Regenerative Braking and Yaw Dynamics Optimal Control in Hybrid Vehicles

Paolo Falcone and Sina KhoshFetrat Pakazad

Department of Signals and Systems,
Chalmers University of Technology, Göteborg, Sweden

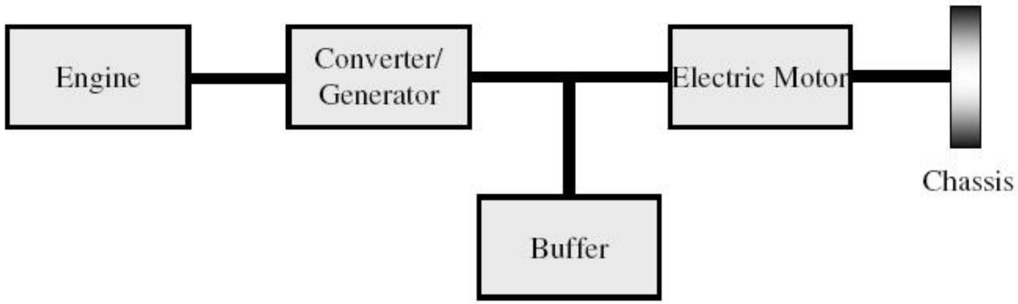
Stefan Solyom

Active Safety Functions, Volvo Car Corporation,
Göteborg, Sweden

Regenerative braking

Hybrid vehicles

- Two possible energy flows in driving and braking



"Series Hybrid Vehicle"

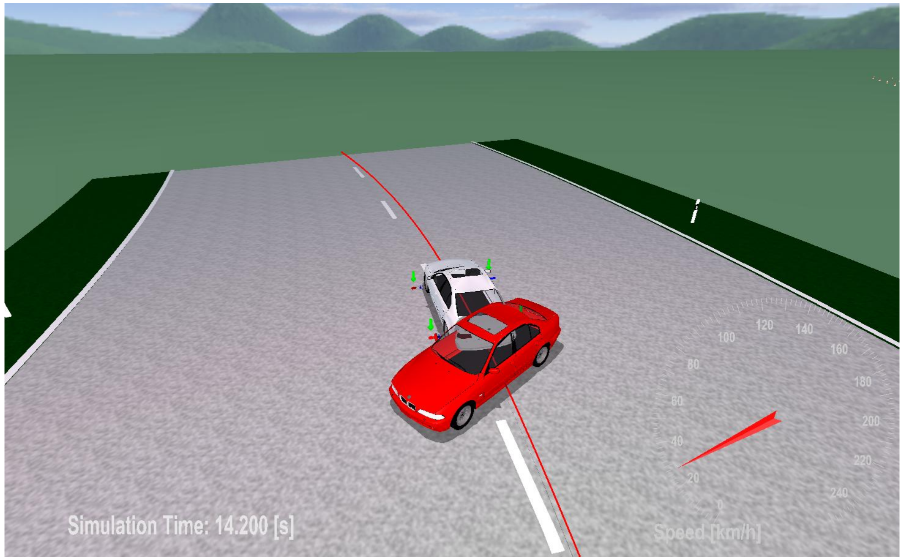
Regenerative braking

Recovering energy by converting vehicle kinetic energy to electric energy

Reduction in fuel consumption

Regenerative braking

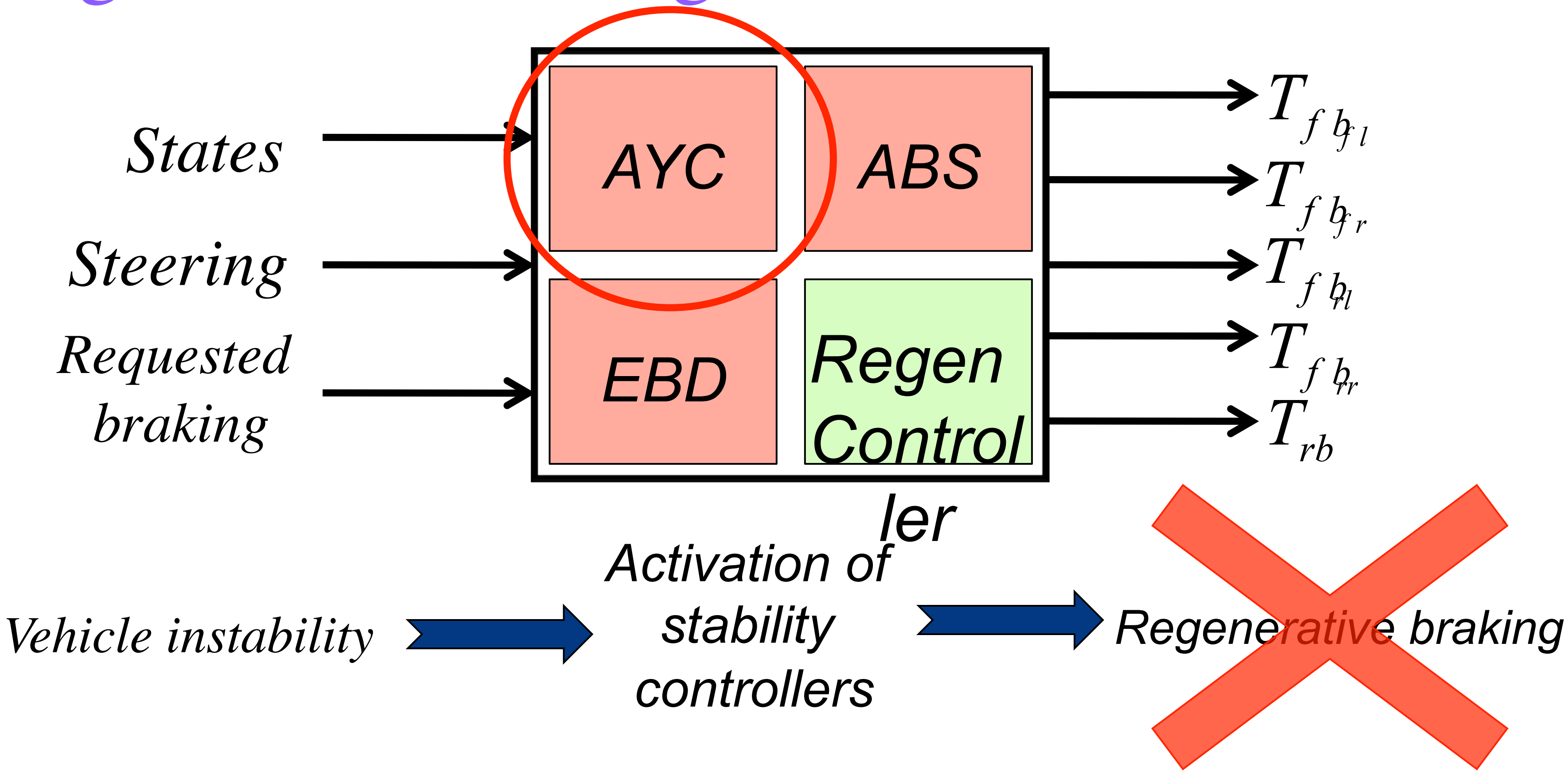
Regenerative braking at the rear axle may induce instability on low-μ surfaces



Given a driver's braking request while *cornering on an icy surface*

maximize the regenerative braking

Regenerative braking



Maximizing the amount of regenerative braking while preserving the vehicle stability and satisfying system constraints.

Outline

- Vehicle modeling
  - Tire modeling
- Model predictive control design
  - Constraints and cost function definition
  - Problem formulation
- Simulation and experimental results
  - F/R braking shift in step μ
- Conclusions

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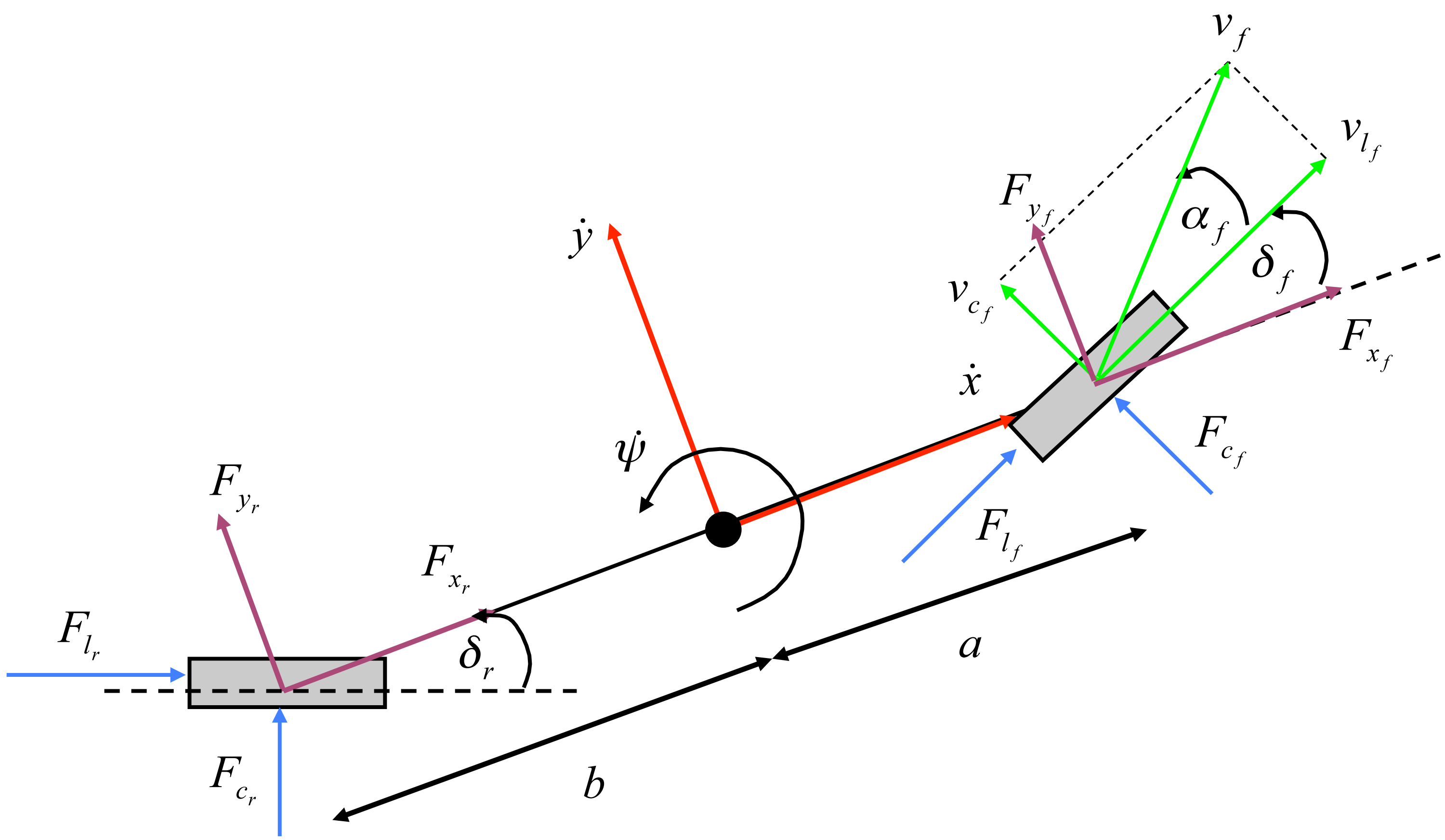
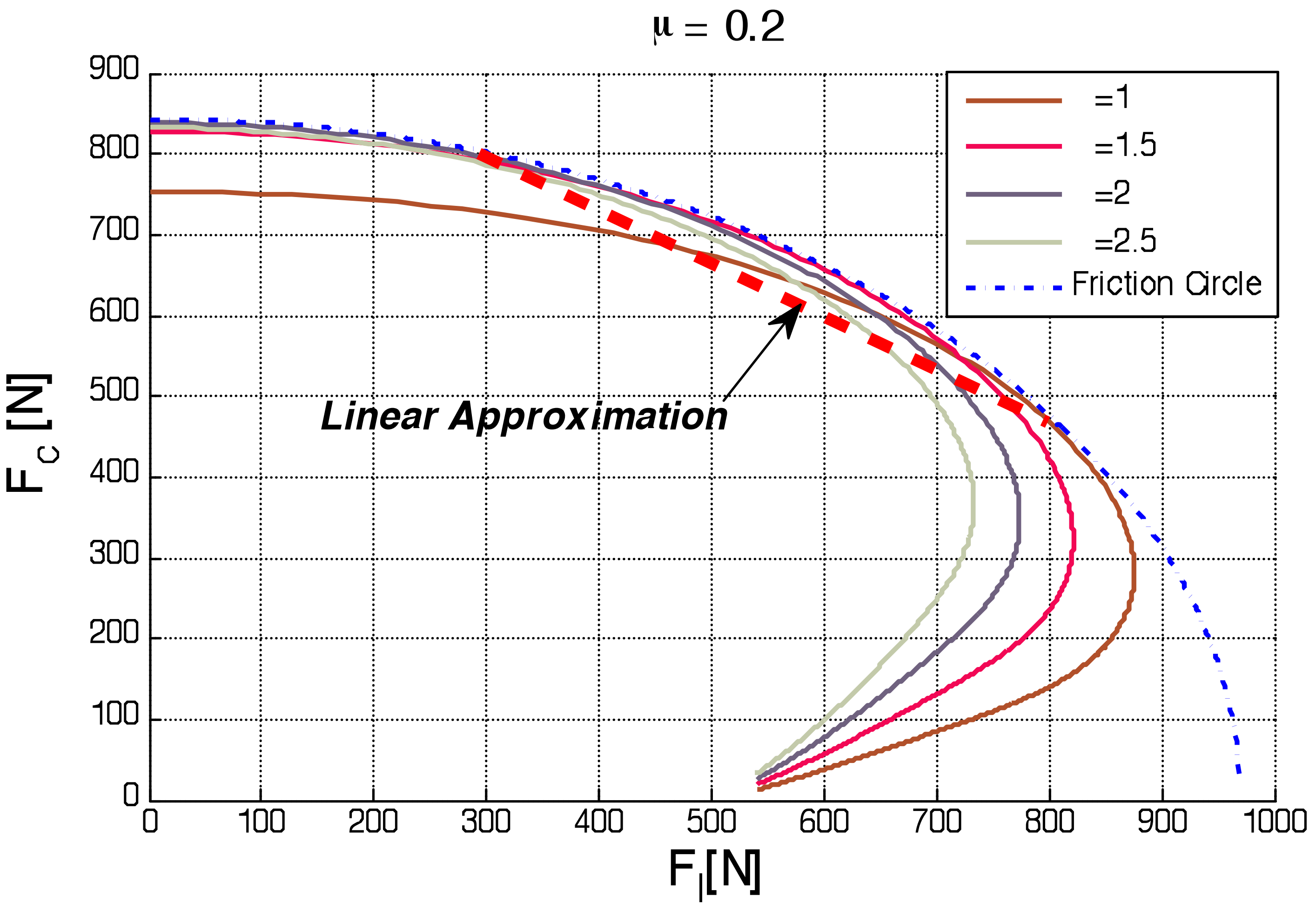
Vehicle modeling. Single track model



Tire modeling

Piecewise Linear approximation : $F_{c_{*,\bullet}} = C_{c_{*,\bullet}} \alpha_{*,\bullet} + D_{c_{*,\bullet}} F_{l_{*,\bullet}}, \; D_{c_{*,\bullet}} = \frac{\partial F_{c_{*,\bullet}}}{\partial F_{l_{*,\bullet}}}.$



Vehicle modeling

Simplified bicycle vehicle model

$$\dot{\xi}(t) = f(\xi(t), u(t), d(t)).$$

where

$$\xi = \begin{bmatrix} \dot{y} & \dot{x} & \dot{\psi} \end{bmatrix},$$

$$u = \begin{bmatrix} F_{fb_f} & F_{fb_r} & F_{rb} \end{bmatrix}, \quad F_{l_{f,\bullet}} = F_{fb_{f,\bullet}},$$

$$d = \delta_f. \qquad F_{l_{r,\bullet}} = F_{fb_{r,\bullet}} + F_{rb}.$$

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Model Predictive Control (MPC)

$$\min_{U} x(N)'Px(N) + \sum_{k=0}^{N-1} x(k)'Qx(k) + u(k)'Ru(k)$$

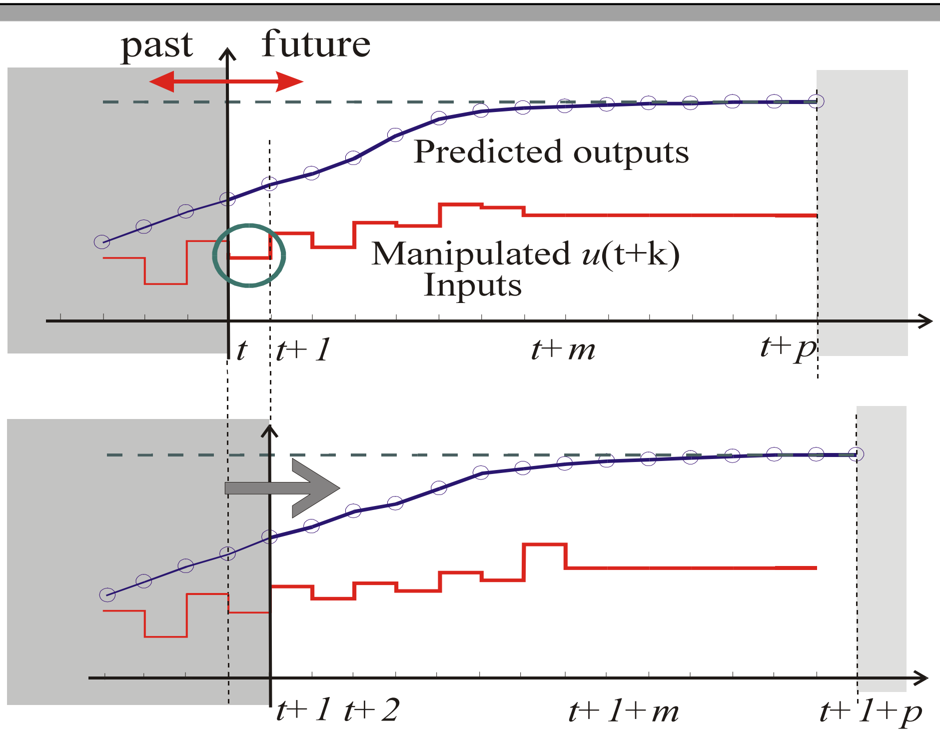
$$\text{subject to} \quad x(k+1) = f\big(x(k), u(k)\big)$$

$$x(k) \in X, u(k) \in U$$

$$g\big(x(k), u(k)\big) = 0$$

Objective function
System dynamics
State/input constraints

Solved in Receding Horizon



1. Optimize at time *t* (new measurements)
2. Only apply the first optimal move *u(t)*
3. Repeat the whole optimization at time *t +1*
4. Optimization using current measurements **Feedback**

Control design

Constraints:

- Vehicle Dynamics Constraints:

$$\xi_{k+1,t} = A_t \xi_{k,t} + B_t u_{k,t} + d_{k,t},$$

$$u_{k,t} = u_{k-1,t} + \Delta u_{k,t},$$

$$\eta_{k,t} = h(\xi(k)) = [0\ 0\ 1]\xi_{k,t}.$$

- Input constraints:
  - Maximum regenerative braking.
  - Total delivered force: $F_D = F_{fb} + F_{rb}$
  - Input rate.
  - 70/30 constraint on friction braking: $F_{fb} = F_{fb_f} + F_{fb_r}$, with $\begin{cases} F_{fb_f} = 0.7 F_{fb} \\ F_{fb_r} = 0.3 F_{fb} \end{cases}$

Control design

- Problem Formulation:
  - Constraints:
    - Constraint on the yaw rate tracking error:

$$E_{\min} - \varepsilon \leq \dot{\psi} - \dot{\psi}_{ref} \leq E_{\max} + \varepsilon$$

  - Cost Function:

$$J(\xi_t, \Delta u_t, \varepsilon) = \sum_{i=1}^{H_p} \left\| \eta_{t+i,t} - \eta_{ref_{t+i,t}} \right\|_Q^2 + \sum_{i=0}^{H_u-1} \left\| \Delta u_{t+i,t} \right\|_R^2 + \sum_{i=0}^{H_u-1} \left\| u_{t+i,t} \right\|_S^2 + \rho \varepsilon^2 .$$

Control design

- Problem Formulation:

$$\min_{\Delta u_t, \varepsilon} J(\xi_t, \Delta u_t, \varepsilon)$$

$$subj.to$$

Vehicle dynamics	$\xi_{k+1,t} = A_t \xi_{k,t} + B_t u_{k,t} + d_{k,t},$ $\eta_{k,t} = h(\xi(k)) = [0\ \ 0\ \ 1]\xi_{k,t}.$ $k = t,\ldots,t+H_p,$
Input constraints	$u_{k,t} = u_{k-1,t} + \Delta u_{k,t},$ $u_{\min} \le u_{k,t} \le u_{\max},$ $F_{D_{k,t}} = \begin{bmatrix}1 & 1 & 1\end{bmatrix} u_{k,t},$ $F_{fb_f} = 2.3 F_{fb_r}$ $\Delta u_{\min} \le \Delta u_{k,t} \le \Delta u_{\max},$
Additional constraints	$A\Delta u_{k,t} \le b_1 + \varepsilon,$ $-A\Delta u_{k,t} \le -b_2 - \varepsilon,$ $k = t,\ldots,t+H_c-1,$ $\xi_{t,t} = \xi(t).$

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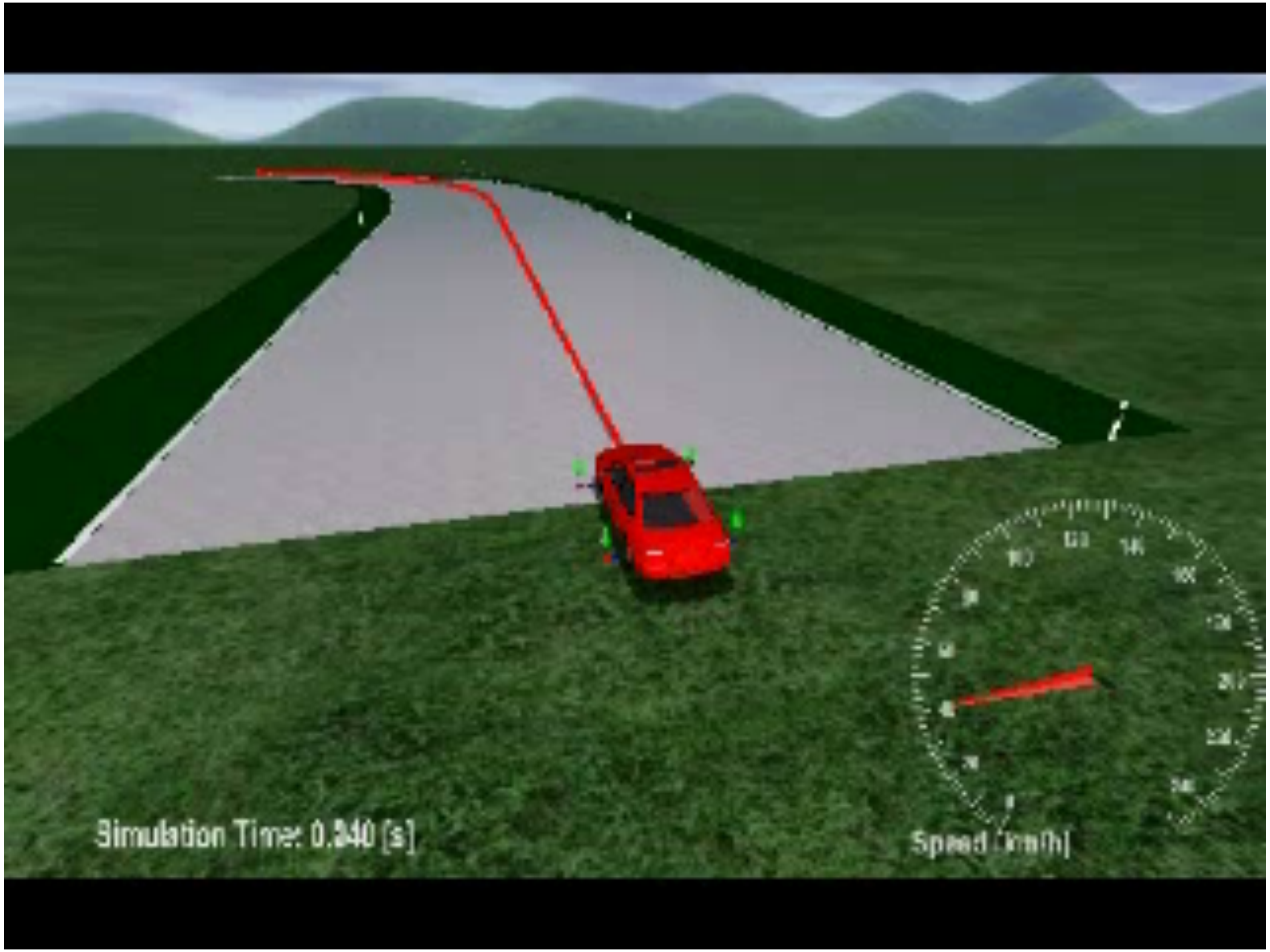
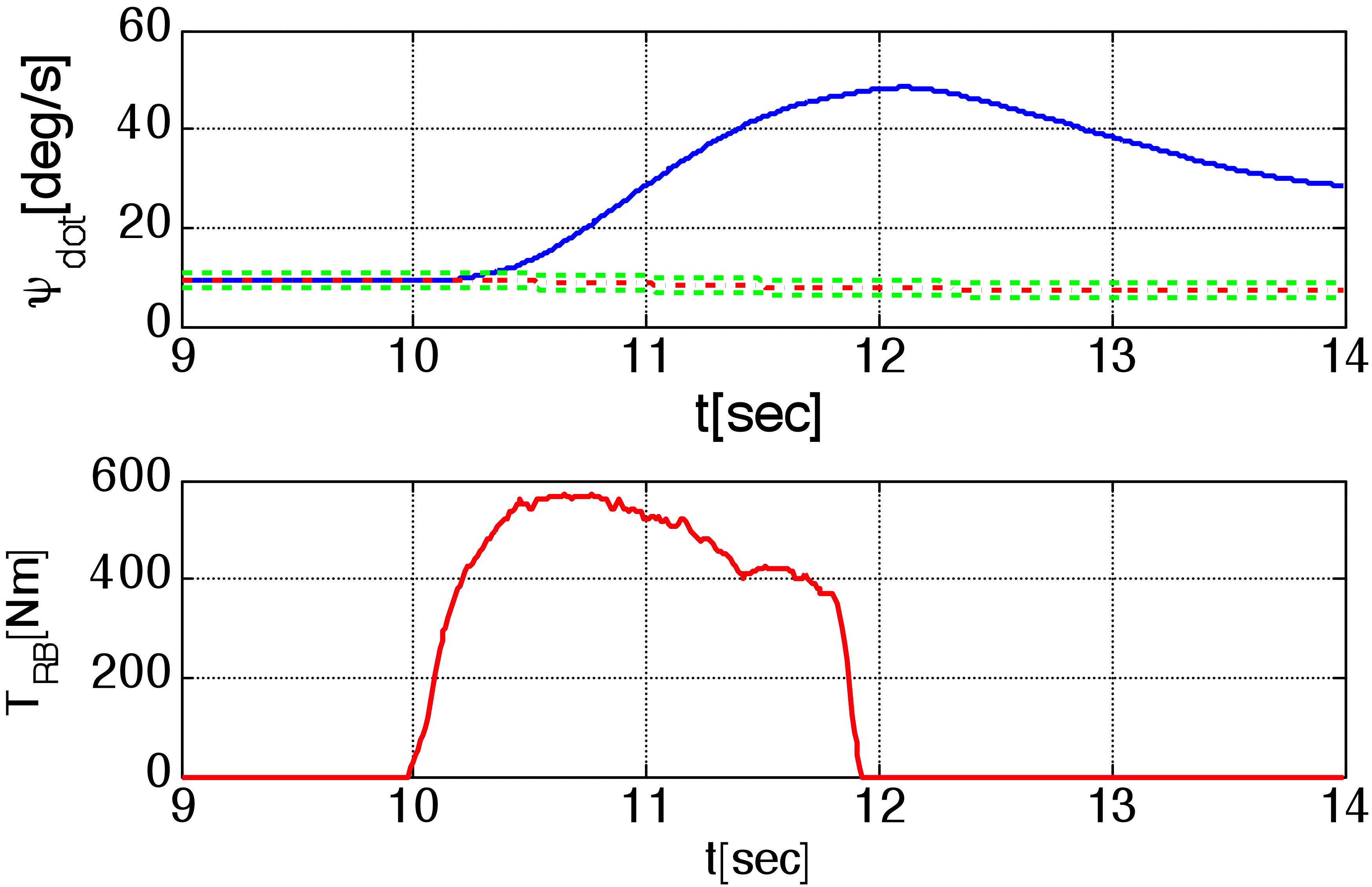
Simulation and Experimental Results:

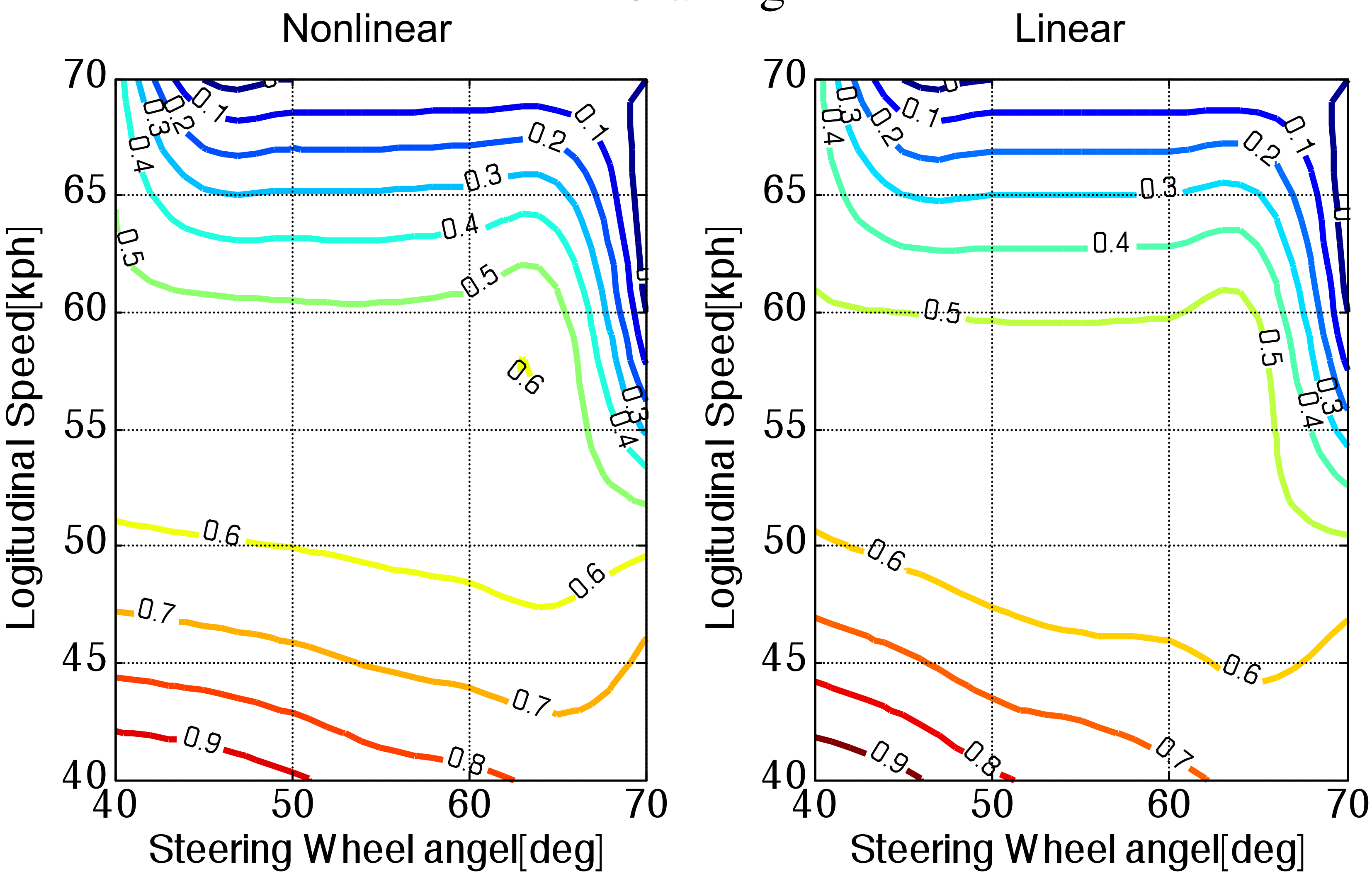


Simulation and Experimental Results:



Nonlinear vs linear MPC controllers

Percentage of requested braking delivered through regenerative braking

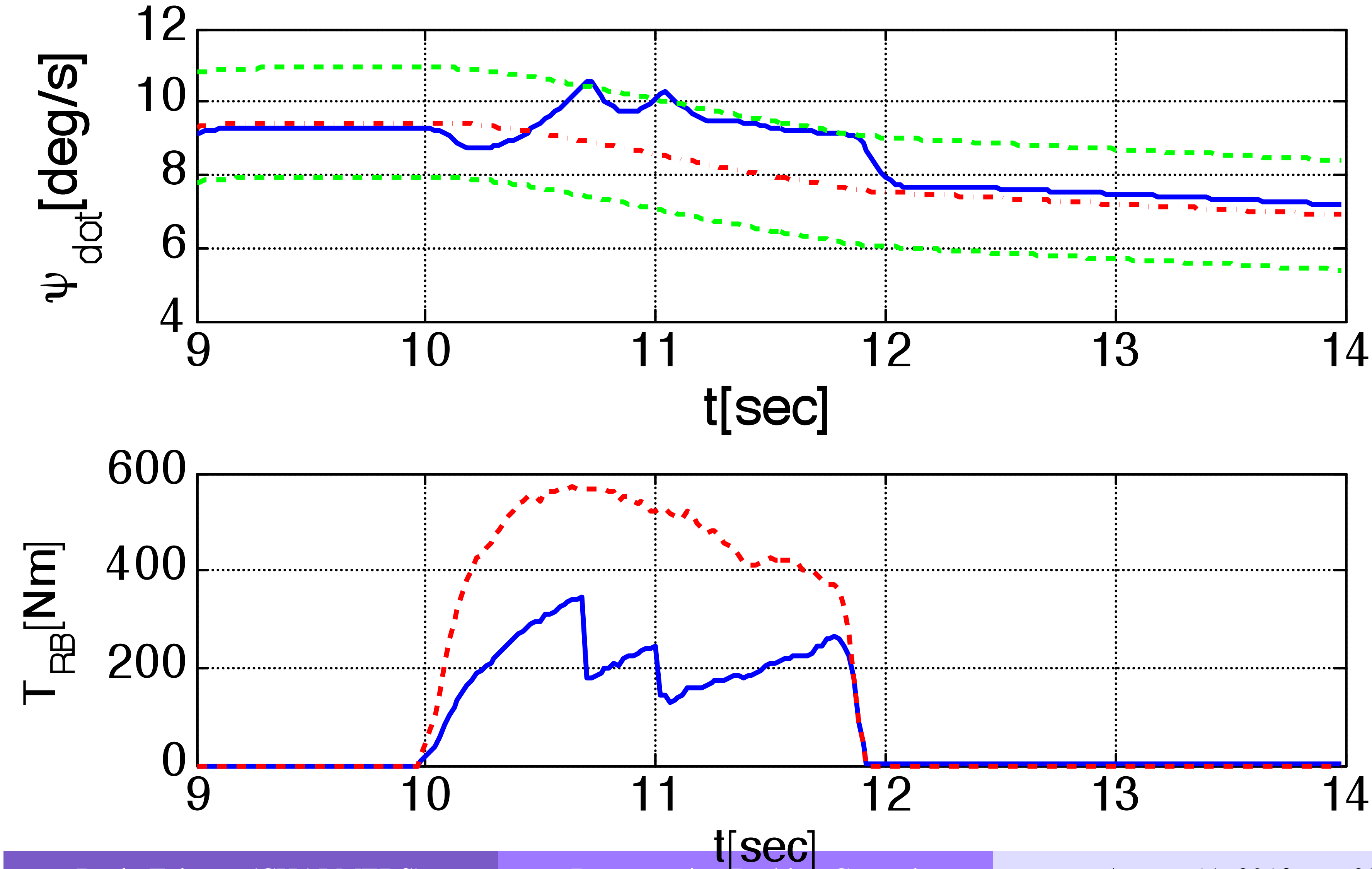


Simulation results. Nonlinear bicycle model

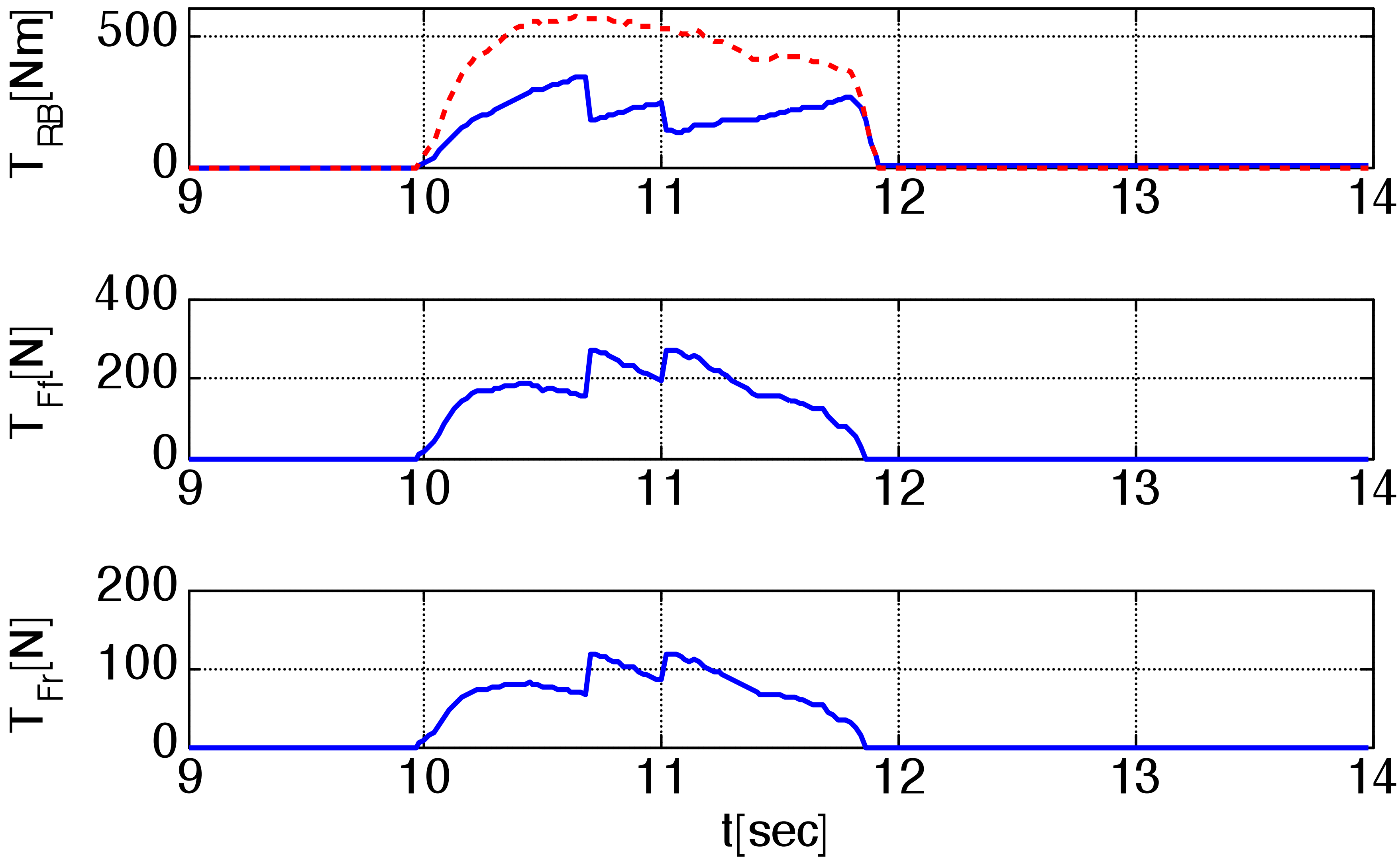
Curve at 50 [Kph] with

40 [deg] Steering wheel angle

50 [Kph]. Yaw rate and regen braking torque



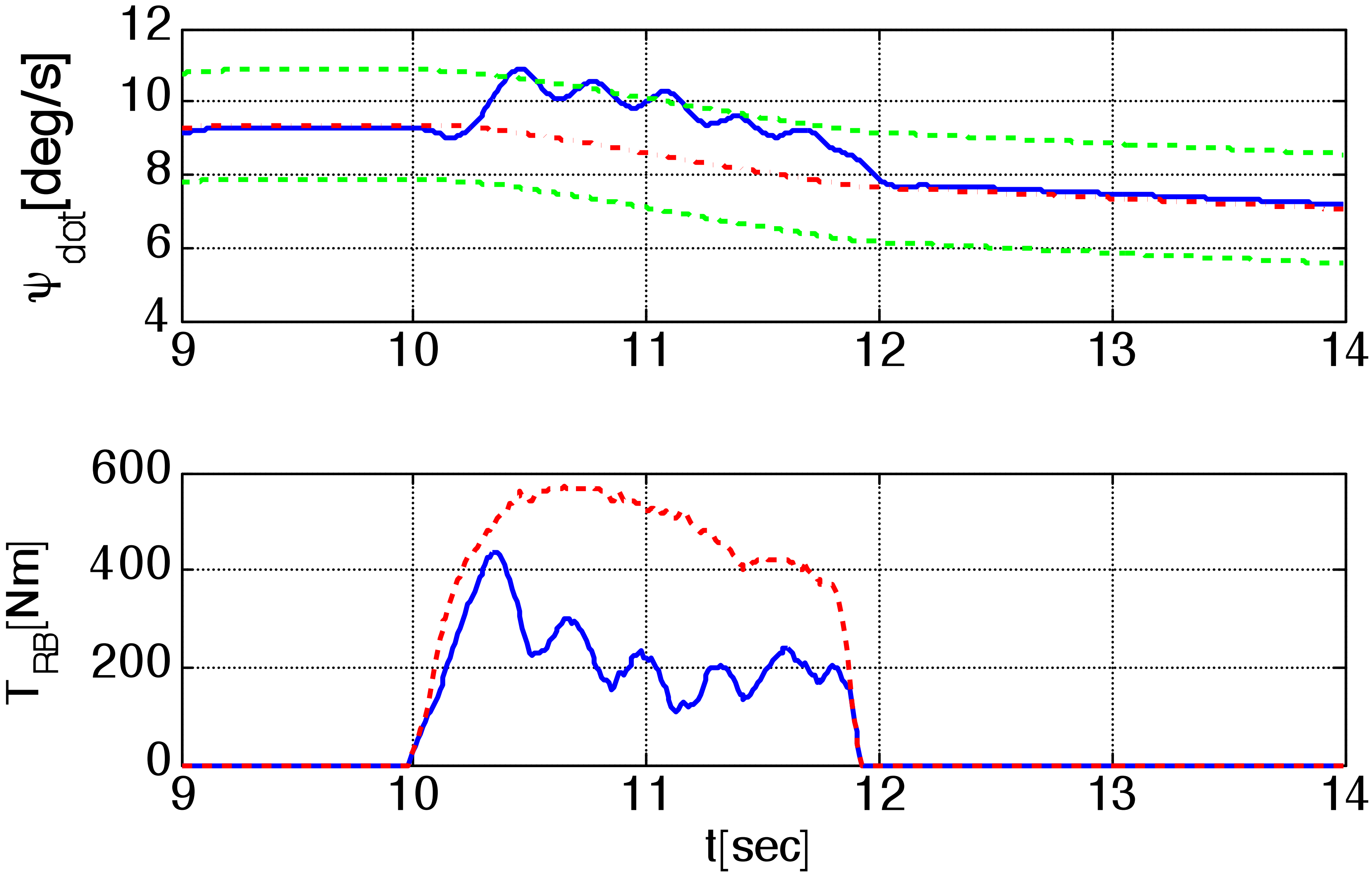
50 [Kph] Braking Torques



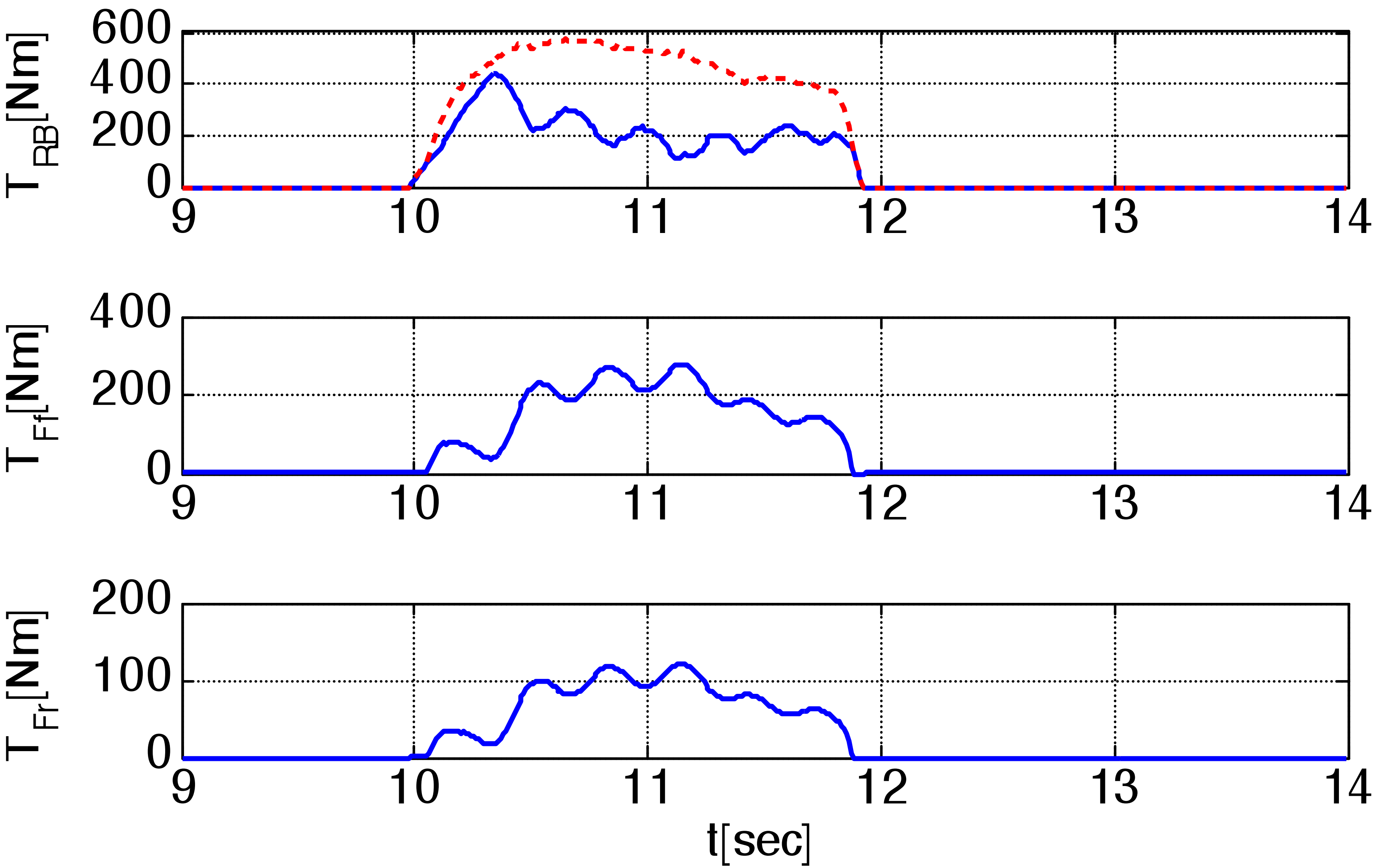
Simulation results. Simplified bicycle model

Curve at 50 [Kph] with
40 [deg] Steering wheel angle

50 [Kph]. Yaw rate and regen braking torque



50 [Kph]. Braking torques

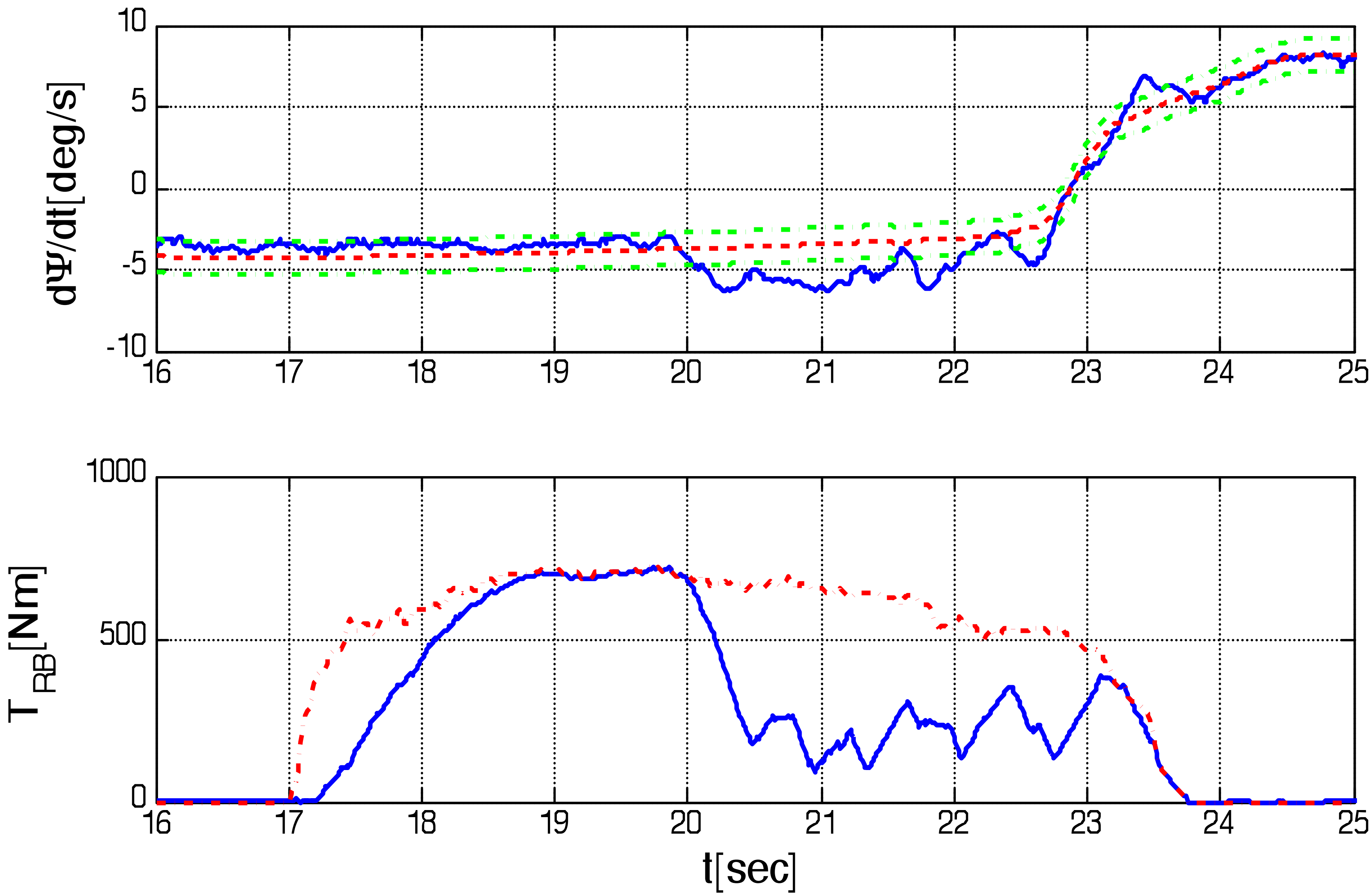


Experimental results

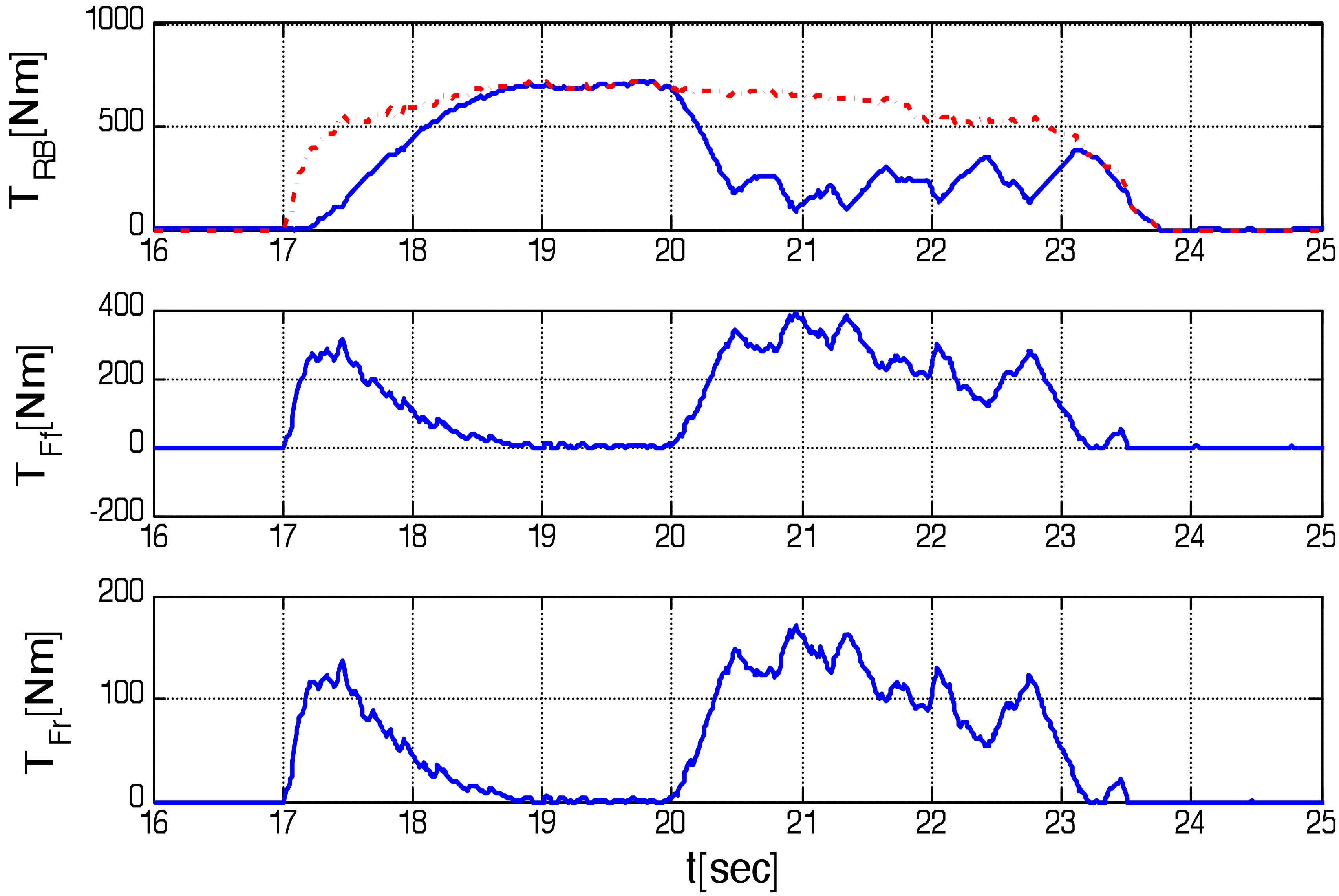
Curve at 60 [Kph] with

50 [deg] Steering wheel angle

60 [Kph]. Yaw rate and regen braking torque



60 [Kph] Braking Torques



Conclusions

- Model based approach to regenerative braking control
- Regenerative braking optimized while preserving vehicle stability
- Higher priority active safety systems activation minimized