PhD Course on Vehicle Dynamics Control. Adaptive Cruise Control

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CHALMERS

Lecture content

1. Main Concepts in ACC
2. Operating modes
3. Control design in vehicle following mode
    1. requirements
    2. constant spacing
    3. constant time gap
4. Transition logics

Note: The following notes have been extracted from

- "Vehicle Dynamics and Control" by R. Rajamani

Lecture content

1. Main Concepts in ACC

Main concepts and intro

ACC objectives

1. Maintaining a constant vehicle longitudinal speed in absence of preceding vehicles
2. Maintaing a "safe" distance from the preceding (slower) vehicle, if any

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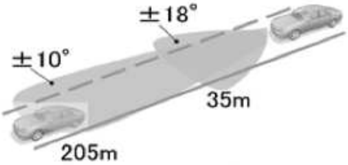

Sensors

1. Speed sensor (odometer)
2. Radar
    - Range through reflections
    - Range rate through doppler effect

Note. The ACC is an “autonomous” system. I.e., no wireless

Main concepts and intro

- First introduced in Japan in early nineties

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Main concepts and intro

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- Originally thought as a "comfort and convenience" system
- According statistics (over 90% highways accident cause by human errors[1]) may impact safety as well
- Basis of many automated driving systems available on the market

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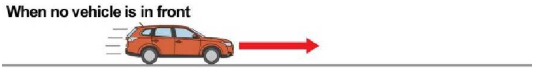
2. Operating modes

Operating modes

Modes

1. Speed control



Operating modes

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...+ logics for

1. “smoothly” switching between the two modes

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Modes

1. Speed control
2. Vehicle following. I.e., maintaing a "safe" distance from the preceding (slower) vehicle, if any

...+ logics for

1. "smoothly" switching between the two modes
2. handling, e.g., cut-in and cut-out maneuvers

Lecture content

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3. Control design in vehicle following mode
    1. requirements

Vehicle following. Control requirements

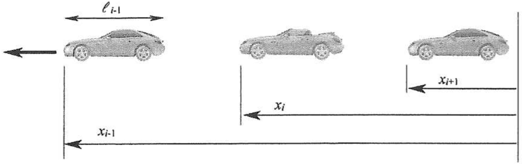
Individual vehicle stability

Define the spacing error as

$$\delta_i = x_i - x_{i-1} + L_{des}.$$

The ACC provide individual vehicle stability if

$$\ddot{x}_{i-1} \to 0 \ \Rightarrow \ \delta_i \to 0$$

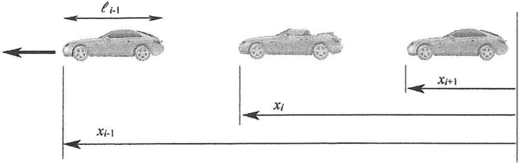


Vehicle following. Control requirements

String stability

The *string stability* property implies that, during velocity transients, the non-zero spacing errors do not amplify toward the tail of a string of ACC vehicles[a]

[a]Swaroop, 1995, Swaroop and Hedrick, 1996



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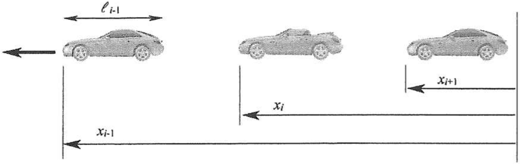

Individual vehicle stability is trivial. ***We will focus on string stability***

Vehicle following. Vehicle model

Assumptions

- Two level hierarchical control
- Upper level calculates a desired acceleration to meet the control requirements
- Lower level calculates the engine and brake low level control inputs

Hence, model the i-th vehicle as either a double integrator

$$\ddot{x}_i = u_i,$$

or as

$$\ddot{x}_i = \frac{e^{-s\tau}}{a + sT} u_i,$$

where $u_i = \ddot{x}_{i_{des}}$. Typically,

$$-5m/s^2 \leq \ddot{x} \leq 2m/s^2$$

Vehicle following. String stability

Definition

Define

$$H(s) = \frac{\delta_i(s)}{\delta_{i-1}(s)}.$$

The chain of ACC vehicles is string stable if[a]

1. $\|H(s)\|_\infty \leq 1$

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Intuitively,

1. Condition 1 guarantees that $\|\delta_i\|_2 \leq \|\delta_{i-1}\|_2$

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2. Condition 2 implies that the *steady state* spacing errors have the same sign

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More rigorous explanation follows

...short detour to norms for signals and systems

Definitions (signals)

Consider a signal $u(t): \; t \in [-\infty, \infty] \rightarrow u \in \mathbb{R}$. Define the following norms

1. **1-Norm** $\|u\|_1 = \int_{-\infty}^{\infty} |u(t)| dt$

Definitions (systems)

Consider a linear, time-invariant, causal system $y = g * u$, where g is the impulse response and $G = \mathscr{L}(g)$

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Useful results on gains[2]

2-norm/2-norm gain

Consider the system $y = g * u$, with $G = \mathscr{L}(g)$.

$$\|G\|_\infty = \sup \frac{\|y\|_2}{\|u\|_2}$$

[2] Doyle, Francis, Tannenbaum, 1992

Useful results on gains

2-norm/2-norm gain

Consider the system $y = g * u$, with $G = \mathscr{L}(g)$.

$$\|G\|_\infty = \sup \frac{\|y\|_2}{\|u\|_2}$$

Proof.

By the Parseval's theorem

$$\begin{aligned}
\|y\|_2^2 &= \|Y\|_2^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} |G(j\omega)|^2 |U(j\omega)|^2 d\omega \\
&\leq \|G\|_\infty^2 \frac{1}{2\pi}\int_{-\infty}^{\infty} |U(j\omega)|^2 d\omega \\
&= \|G\|_\infty^2 \|U\|_2^2 = \|G\|_\infty^2 \|u\|_2^2
\end{aligned}$$

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Show now that $\|G\|_\infty$ is the least upper bound on the 2-norm/2-norm gain.

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Show now that $\|G\|_\infty$ is the least upper bound on the 2-norm/2-norm gain.

Choose u such that $\|u\|_2 = 1$ and show that $\|Y\|_2^2 = \|G\|_\infty^2$ $\square$

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Proof.

Apply the Cauchy-Schwarz inequality

$$\begin{aligned}
|y(t)| &= \left| \int_{-\infty}^{\infty} g(t-\tau)u(\tau)d\tau \right| \\
&\leq \left(\int_{-\infty}^{\infty} g(t-\tau)^2 d\tau \right)^{1/2} \left(\int_{-\infty}^{\infty} u(\tau)^2 d\tau \right)^{1/2} \\
&= \|g\|_2 \|u\|_2 = \|G\|_2 \|u\|_2
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Hence $\|y\|_\infty \leq \|G\|_2 \|u\|_2$

$\square$

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Proof follows the same steps as before $\square$

Useful results on gains

∞-norm gain/2-norm

Consider the system $y = g * u$, with $G = \mathscr{L}(g)$.

$$\frac{\|y\|_2}{\|u\|_\infty} = \infty$$

Useful results on gains

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Proof.

Choose a sinusoidal input signal with frequency ω, such that ω is not a zero of G. Hence $\|u\|_\infty = 1$ and $\|y\|_2^2$ is unbounded $\square$

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Consider the system $y = g * u$, with $G = \mathscr{L}(g)$.

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Proof.

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Useful results on gains[2]

If $g(t) > 0 \ \forall t \geq 0$ then $\|g\|_1 = \|G\|_\infty$

[2] Swaroop, 1995

Useful results on gains

If $g(t) > 0 \ \forall t \geq 0$ then $\|g\|_1 = \|G\|_\infty$

Proof.

Be $\gamma_p = \sup \dfrac{\|y\|_p}{\|u\|_p}$ for a induced p-norm. Since $\dfrac{\|y\|_p}{\|u\|_p} \leq \|g\|_1$,

$$|G(0)| \leq \|G(j\omega)\|_\infty \leq \gamma_p \leq \|g\|_1.$$

If $g(t) > 0$ then

$$|G(0)| = \left| \int_0^\infty g(\tau) d\tau \right| \leq \int_0^\infty |g(\tau)| d\tau = \|g\|_1$$

$\square$

In summary

Table: System gains

	$\|u\|_2$	$\|u\|_\infty$
$\|y\|_2$	$\|G\|_\infty$	∞
$\|y\|_\infty$	$\|G\|_2$	$\|g\|_1$

Moreover, if $g(t) > 0 \ \forall t \geq 0$ then $\|g\|_1 = \|G\|_\infty$

Vehicle following. String stability

Definition

Define

$$H(s) = \frac{\delta_i(s)}{\delta_{i-1}(s)}.$$

The chain of ACC vehicles is string stable if[a]

1. $\|H(s)\|_\infty \leq 1$

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The main objective is to obtain

$$\|\delta_i\|_\infty \leq \|\delta_{i-1}\|_\infty,$$

i.e., $\|h\|_1 \leq 1$. This is equivalent to $\|H\|_\infty \leq 1$, with the additional condition $h(t) > 0, \ \forall t \geq 0$.

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Constant spacing control design

Define the *inter-vehicle spacing* as

$$\epsilon_i = x_i - x_{i-1} + \ell_{i-1},$$

where ℓ_{i-1} is the length of the $i-1$-th vehicle.

Constant spacing control design

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where L_{des} is the desired distance and includes ℓ_{i-1}.

Consider a double integrator model for the vehicle and a *linear PD controller*

$$\ddot{x}_i = -k_p\delta_i - k_v\dot{\delta}_i$$

Constant spacing control design

Differentiate twice the spacing error

$$\ddot{\delta}_i = \ddot{x}_i - \ddot{x}_{i-1} = -k_p \delta_i - k_v \dot{\delta}_i + k_p \delta_{i-1} + k_v \dot{\delta}_{i-1}$$

Constant spacing control design

Differentiate twice the spacing error

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Rearranging leads to the closed-loop error dynamics

$$\ddot{\delta}_i + k_v\dot{\delta}_i + k_p\delta_i = k_p\delta_{i-1} + k_v\dot{\delta}_{i-1},$$

corresponding to the transfer function

$$H(s) = \frac{\delta_i(s)}{\delta_i(s)} = \frac{k_p + k_v s}{s^2 + k_v s + k_p}$$

Constant spacing control design

Differentiate twice the spacing error

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$$H(s) = \frac{\delta_i(s)}{\delta_i(s)} = \frac{k_p + k_v s}{s^2 + k_v s + k_p}$$

Problem. Find $k_p,\ k_v$ such that

$$\|H\|_\infty \leq 1$$

Constant spacing control design

Solution. For individual vehicle stability, it must be $k_v,\ k_p > 0$.

Constant spacing control design

Solution. For individual vehicle stability, it must be $k_v,\ k_p > 0$.

Rewrite $H(s)$ as

$$H(s) = \underbrace{\frac{k_p}{s^2 + k_v s + k_p}}_{H_1(s)} \underbrace{\left(\frac{k_v}{k_p} s + 1\right)}_{H_2(s)}$$

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In order to have $\|H_1\|_\infty < 1$, the damping must be larger than 0.707, i.e.,

$$\frac{k_v}{2\sqrt{k_p}} \geq 0.707 \ \Rightarrow\ k_v \geq 1.4141\sqrt{k_p}$$

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H_2 has to be below one up to the resonant frequency $\sqrt{k_p}$. Hence,

$$\frac{k_p}{k_v} \geq \sqrt{k_p} \ \Rightarrow \ \sqrt{k_p} > k_v$$

Constant spacing control design

Solution. In conclusion, the following conditions have to be satisfied

$$k_v \geq 1.4141\sqrt{k_p},\ \sqrt{k_p} > k_v,\ k_p, k_v > 0$$

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String stability can't be achieved with a PD controller based on constant spacing policy

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Question. Can string stability be achieved with any other ***linear controller***?

Constant spacing control design

Solution. In conclusion, the following conditions have to be satisfied

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String stability can't be achieved with a PD controller based on constant spacing policy

Question. Can string stability be achieved with any other ***linear controller***?

Answer. No, unless. . . .

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Consider a double integrator model for the vehicle and the control law

$$u_i = -\frac{1}{h}\left(\dot{\epsilon}_i + \lambda\delta_i\right)$$ [2]

[2] Chien, 1993

Constant time gap control design

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Define the *spacing error* as

$$\delta_i = x_i - x_{i-1} + L_{des},$$

where $L_{des} = \ell_{i-1} + h\dot{x}_i$ and h is the time gap

Consider a double integrator model for the vehicle and the control law

$$u_i = -\frac{1}{h}\left(\dot{\epsilon}_i + \lambda\delta_i\right)$$

The error dynamics become

$$\dot{\delta}_i = -\lambda\delta_i$$

Constant time gap control design

Analyze the string stability property of the CTG policy

Constant time gap control design

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Combine the first order vehicle model and the control law $u_i = -\frac{1}{h}\left(\dot{\epsilon}_i + \lambda \delta_i\right)$. Obtain

$$\tau \dddot{x}_i + \ddot{x}_i = -\frac{1}{h}\left(\dot{\epsilon}_i + \lambda \delta_i\right)$$

Constant time gap control design

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Differentiate twice the spacing error $\delta_i = \epsilon_i + h\dot{x}_i$ and replace $\dddot{x}_i$ to obtain

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Solve for ϵ_i and replace in $\delta_i - \delta_{i-1} = \epsilon_i - \epsilon_{i-1} + h\dot{\epsilon}_i$ to obtain

$$H(s) = \frac{\delta_i}{\delta_{i-1}} = \frac{s+\lambda}{h\tau s^3 + hs^2 + (1+\lambda h) + \lambda}$$

Constant time gap control design

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Problem. Find condition on τ and h such that $\|H\|_\infty \leq 1$

Constant time gap control design

Theorem

$\|H\|_\infty \leq 1$ if and only if $h \geq 2\tau$.

Constant time gap control design

Proof

Consider the transfer function

$$H(s) = \frac{\delta_i}{\delta_{i-1}} = \frac{s+\lambda}{h\tau s^3 + hs^2 + (1+\lambda h) + \lambda}$$

Constant time gap control design

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Consider the transfer function

$$H(s) = \frac{\delta_i}{\delta_{i-1}} = \frac{s+\lambda}{h\tau s^3 + hs^2 + (1+\lambda h) + \lambda}$$

Substitute $s = j\omega$

$$H(s)\,|_{s=j\omega} \; = \frac{j\omega + \lambda}{(\lambda - h\omega^2) + j\omega\,(1 + \lambda h - \tau h\omega^2)}$$

Constant time gap control design

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Calculate

$$|H(s)|^2 = \frac{\omega^2 + \lambda^2}{(\lambda - h\omega^2)^2 + \omega^2\,(1 + \lambda h - \tau h\omega^2)^2}$$

Constant time gap control design

Proof (Cont.)

Imposing $|H(j\omega)| \leq 1$ leads to

$$\omega^2 + \lambda^2 \leq \left(\lambda - h\omega^2\right)^2 + \omega^2 \left(1 + \lambda h - \tau h\omega^2\right)^2$$

Constant time gap control design

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Squaring the terms in parentheses and rearranging

$$\tau^2 h^2 \omega^4 + \left(h^2 - 2\tau h - 2\tau\lambda h^2\right)\omega^2 + \lambda^2 h^2 \geq 0$$

Constant time gap control design

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Study positiveness of $a\omega^4 + b\omega^2 + c$. Rewrite

$$\begin{aligned} a\omega^4 + b\omega^2 + c &= a\left(\omega^4 + 2\frac{b}{2a}\omega^2 + \frac{c}{a}\right) \\ &= a\left[\left(\omega^2 + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a^2}\right] \end{aligned}$$

Constant time gap control design

Proof (Cont.)

Hence $a\omega^4 + b\omega^2 + c > 0$ if

Constant time gap control design

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1. $a,\ b,\ c > 0$
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Constant time gap control design

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Constant time gap control design

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Distinguish the following two cases

1. $b > 0$ corresponds to $h^2 - 2\tau h - 2\lambda\tau h^2 > 0$. Hence

$$h > \frac{2\tau}{1 - 2\lambda\tau}.$$

   For small λ, this is possible if $h > 2\tau$.

Constant time gap control design

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Distinguish the following two cases

1. $b > 0$ corresponds to $h^2 - 2\tau h - 2\lambda\tau h^2 > 0$. Hence
$$h > \frac{2\tau}{1 - 2\lambda\tau}.$$
   For small λ, this is possible if $h > 2\tau$.
2. $b < 0,\ a > 0,\ c > 0$ and $b^2 - 4ac < 0$ corresponds to
$$\left(h^2 - 2\tau - 2\lambda\tau h^2\right)^2 - 4\tau^2 h^4 \lambda^2$$

Constant time gap control design

Proof (Cont.)

2. Simplify to obtain

By relaxing the inequality in $a\omega^4 + b\omega^2 + c > 0$, $h \geq 2\tau$ follows.

$\square$

Constant time gap control design

Proof (Cont.)

2. Simplify to obtain

$$\begin{aligned} \lambda &< \frac{4\tau h - h^2 - 4\tau^2}{8\tau^2 h - 4\tau h^2}, \\ \lambda &< \frac{-\left(2\tau - h\right)^2}{4\tau h\left(2\tau - h\right)}. \end{aligned}$$

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\end{aligned}
$$

   Since $\lambda > 0$, it must be $h > 2\tau$.

By relaxing the inequality in $a\omega^4 + b\omega^2 + c > 0$, $h \geq 2\tau$ follows.

By 1) and 2) also follows that if $h \geq 2\tau$ a λ can be found such that $|H(j\omega)| < 1$. □

Constant time gap control design

Note that

1. The previous condition does not guarantee that $\|H\|_\infty \leq 1$, $h(t) > 0, \ \forall t \geq 0$ hold simultaneously.

Constant time gap control design

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   a) The dominant poles of $H(s)$ should not be complex conjugate

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   a) The dominant poles of $H(s)$ should not be complex conjugate
   b) $H(s)$ should not have positive zeros

Lecture content

1. Main Concepts in ACC
2. Operating modes
3. Control design in vehicle following mode
    1. requirements
    2. constant spacing
    3. constant time gap
4. Transition logics

Transition logics

Transition scenarios

1. Speed control $\longrightarrow$Vehicle following.

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   - Slower vehicle ahead
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Example. A car is initially operating in speed control mode at 30 m/s, when a stalled car is detected 100 m ahead. The parameters of the CTG law are $\lambda = 1,\ h = 1\ s,\ L = 5\ m$.

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$$\begin{aligned} \delta_i &= x_i - x_{i-1} + L + h\dot{x}_i, \\ &= -100 + 5 + 30 = -65, \end{aligned}$$

and the initial relative velocity is $\dot{\epsilon}_i = \dot{x}_i - \dot{x}_{i-1} = 30$.

Transition logics

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1. Speed control $\longrightarrow$Vehicle following. In, e.g.,
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and the initial relative velocity is $\dot{\epsilon}_i = \dot{x}_i - \dot{x}_{i-1} = 30$. According to the CTG law $u = -\frac{1}{h}\left(\dot{\epsilon}_i + \lambda\delta_i\right)$, the
acceleration $u = -1(30 - 65) = \mathbf{35}\ m/s^2$ is demanded.

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A *range-range rate diagram*[2] can be used

[2] Fancher and Bareket, 1994

Transition logics

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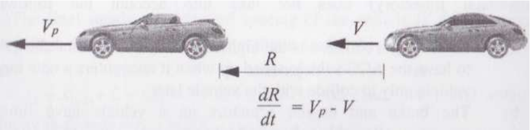
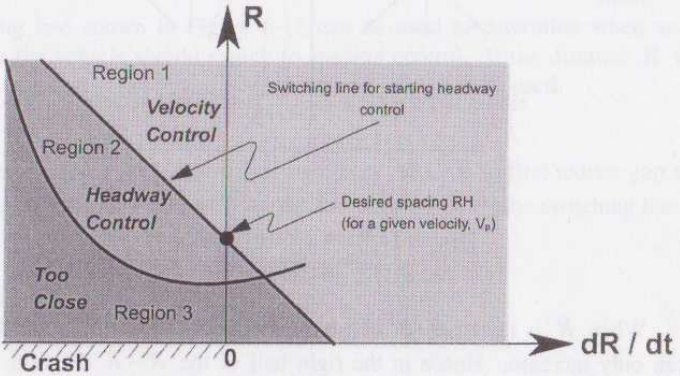
A *range-range rate diagram* can be used

Define range R and range rate $\dot{R}$ as in the picture below



Transition logics

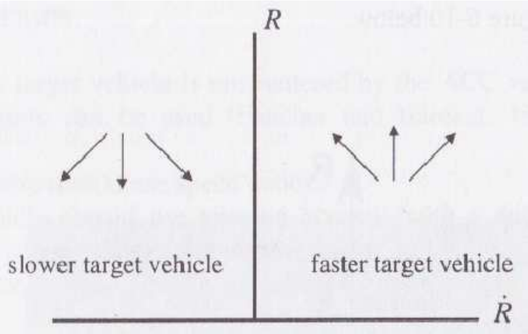
A typical $R - \dot{R}$ diagram is[2]



[2] Fancher and Bareket, 1994

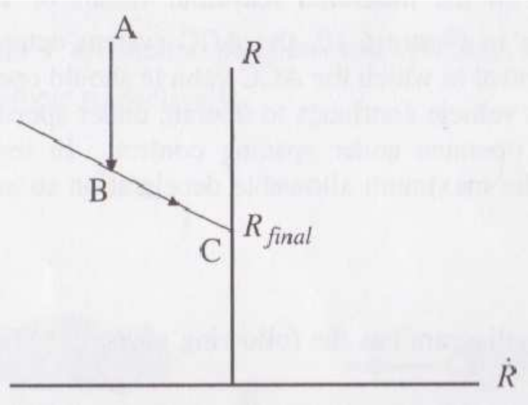
Transition logics

The possible directions of motion are



Transition logics

The line separating the speed and spacing control regions is given by $R = -T\dot{R} + R_{final}$



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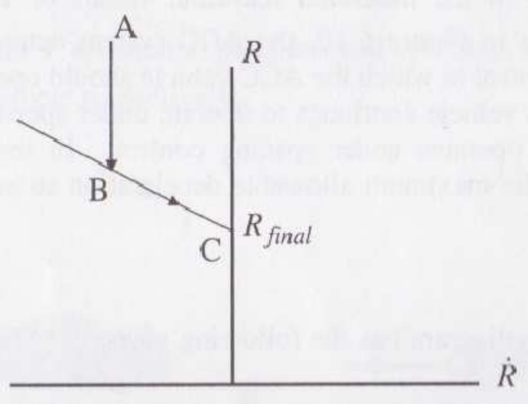
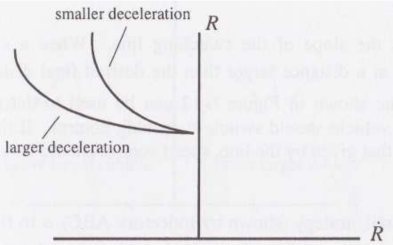


The control law on this transitional trajectory is given by

$$u = -k_p\left(x - R\right) - k_d\left(\dot{x} - \dot{R}\right)$$

Transition logics

During constant deceleration the trajectory in the $R - \dot{R}$ plane is a parabola with equation $R = R_{amn} + \dfrac{\dot{R}^2}{2D}$



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