Robust Vehicle Stability Control with an Uncertain Driver Model

Ashwin Carvalho, G. Palmieri, H. E. Tseng, L. Glielmo, F. Borrelli

Outline

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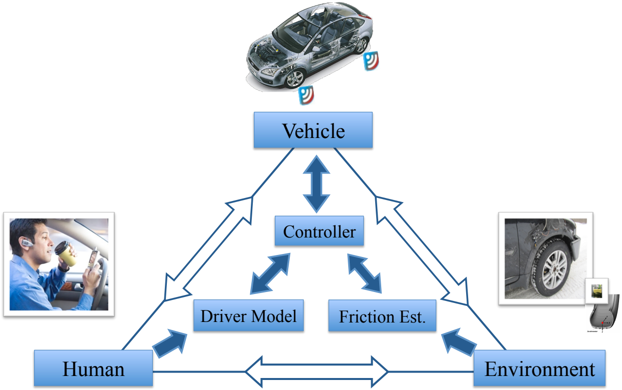
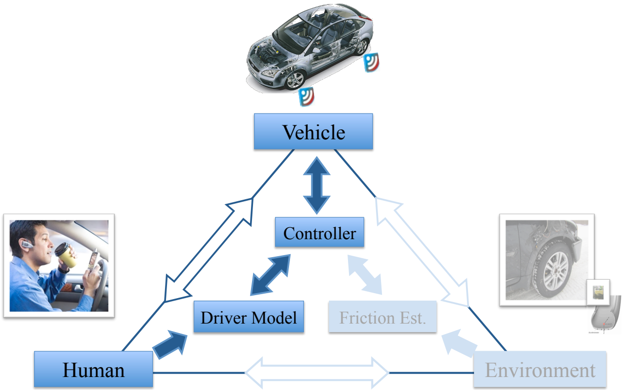
Active Safety: An Integrated Approach



- Driver, vehicle and environment as a closed-loop system, where the driver is the principal actuator.

Active Safety: An Integrated Approach



- In this work:
  - Lateral stability control
  - Robust to uncertainties in model and driver's actions

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Problem Statement

Goal

- Combination of steering and braking corrections
- Avoid front and rear tire force saturation
- Track yaw rate and lateral velocity references

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Systematic approach to address all four challenges in a unified framework

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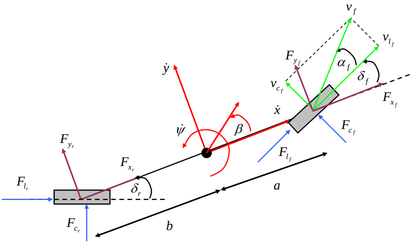
Lateral Dynamics Model

- Classical nonlinear bicycle model:



$$m\ddot{y} = -m\dot{x}\dot{\psi} + 2F_{c_f} + 2F_{c_r},$$
$$I_z\ddot{\psi} = 2aF_{c_f} - 2bF_{c_r} + M.$$

- States: $\dot{y}$, $\dot{\psi}$
- Inputs: δ_f, M

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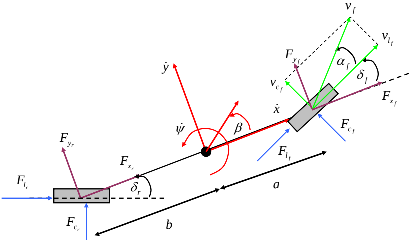

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- Cornering forces F_{c_f} and F_{c_r} are given by the Pacejka tire model:

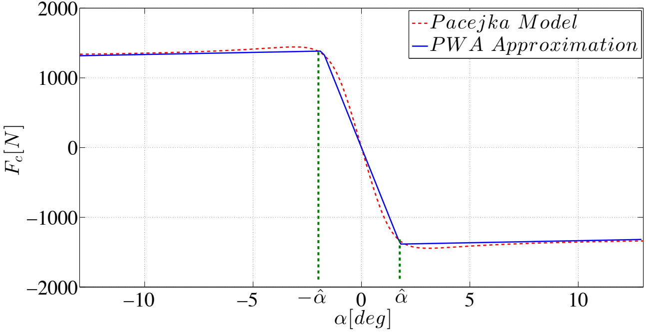
$$F_{c_\star} = f_{c_\star}(\alpha_\star, \sigma_\star, F_{z_\star}, \mu_\star), \qquad \alpha_\star = g_\star(\dot{y}, \dot{\psi}, \delta_f)$$

- Nonlinear functions of vehicle's states and inputs

Cornering Force: PWA Approximation

- Approximate nonlinear function by a piecewise affine (PWA) function:

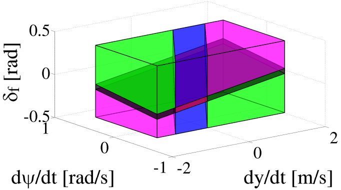
$$F_c(\alpha_\star) = \begin{cases} c_s\alpha_\star + (c_l + c_s)\hat{\alpha}_\star, & \text{if } \alpha_\star \le -\hat{\alpha}_\star \\ -c_l\alpha_\star, & \text{if } -\hat{\alpha}_\star \le \alpha_\star \le \hat{\alpha}_\star \\ c_s\alpha_\star - (c_l + c_s)\hat{\alpha}_\star, & \text{if } \alpha_\star \ge \hat{\alpha}_\star \end{cases}$$



PWA Bicycle Model

- Hybrid model with 9 modes:

$$\begin{bmatrix} \ddot{y} \\ \ddot{\psi} \end{bmatrix} = A_i \begin{bmatrix} \dot{y} \\ \dot{\psi} \end{bmatrix} + B_i \begin{bmatrix} \delta_f \\ M \end{bmatrix} + f_i$$
$$(i = 1, 2, ..., 9)$$



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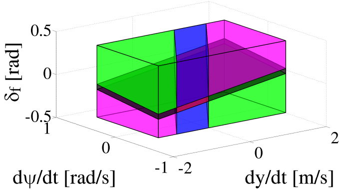

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$$\delta_f = \delta_d + \delta_{AFS}$$

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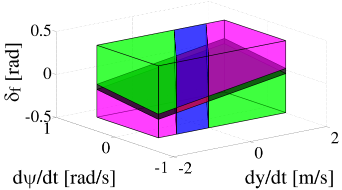

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- In discrete–time,

$$\begin{aligned} z(k+1) = & A_i^d z(k) + B_i^d u(k) + W_i^d \delta_d(k) + f_i^d \\ & (z, u, \delta_d) \in \mathcal{Q}_i \qquad (i = 1, 2, ..., 9) \\ & z := [\dot{y}, \dot{\psi}]^T \qquad u := [\delta_{AFS}, M]^T \end{aligned}$$

Advantages of Including Driver's Input in Model

- Impose bounds directly on δ_{AFS} (as opposed to δ_f)
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Remark

To be robust to the uncertainty in δ_d, we need to construct state–dependent bounds on δ_d. That is,

$$\delta_d \in \mathcal{W}_z(z)$$

Uncertain Driver Model

- Steady–state relationship between $\dot{\psi}$ and δ_f:

$$\dot{\psi}_{ss} = \frac{v_x}{l(1 + v_x^2/v_{ch}^2)}\delta_{f,ss} =: \frac{\delta_{f,ss}}{K_{\psi,ss}} =: G_{\psi,ss}\delta_{f,ss}$$

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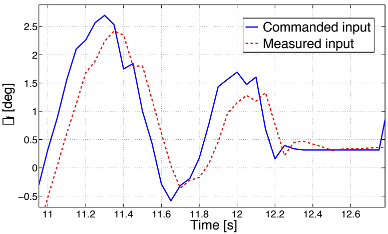
- To obtain $\mathcal{W}_z(\cdot)$, we assume that the actual value of δ_d lies in an interval centered at $\delta_{d,ss}$:

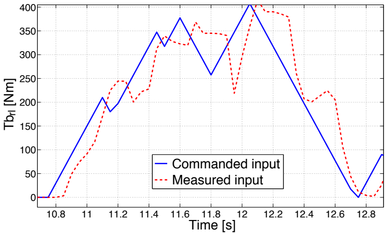
$$\begin{aligned} |\delta_d - K_{\psi,ss}\dot{\psi}| &\leq \epsilon > 0 \\ |\delta_d| &\leq \delta_{d,max} \end{aligned}$$

- Parameters K_ψ and ϵ chosen based on experimental data

Uncertainty in Actuation



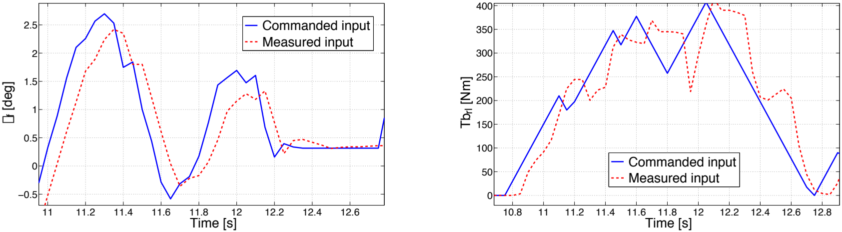
(a) Delay



(b) Unmodeled dynamics

Figure: Actuation errors

Uncertainty in Actuation



(a) Delay (b) Unmodeled dynamics

Figure: Actuation errors

- Introduce additive input–dependent uncertainty w_u in the model

$$w_u \in \mathcal{W}_u(u)$$

Vehicle Model: Summary

Hybrid model with 9 modes

$$
\begin{aligned}
z(k+1) = &A_i^d z(k) + B_i^d u(k) + W_i^d \delta_d(k) + f_i^d + B_i^d w_u \\
&(z, u, \delta_d) \in \mathcal{Q}_i \quad (i = 1, 2, ..., 9) \\
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- Modes are state, input and disturbance dependent
- Driver's input explicitly included
- Both, input and state–dependent uncertainties are present

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Robust Control Design

Objective

Keep front and rear tires in linear region (mode 1) *for all admissible values* of δ_d and w_u

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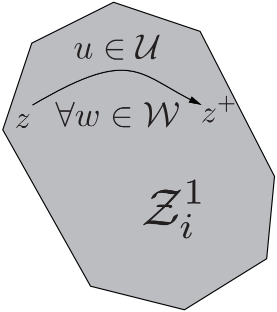
Reaching a specified **target set** and **staying** within the target set **for all admissible values of the disturbance variables** can be formalized using robust reachability framework

Mode 1 Robust Control Invariant (RCI) Set

Definition

The *RCI set* $\mathcal{Z}_i^1 \subseteq \mathcal{P}_1 = \mathrm{Proj}_z(Q_1)$ *associated with mode 1* is defined as:

$$\mathcal{Z}_i^1 := \{z \in \mathcal{P}_1 : \forall \delta_d \in \mathcal{W}_z(z), \exists u \text{ such that}$$
$$(z, u, \delta_d) \in \mathcal{Q}_1, z^+ \in \mathcal{Z}_i^1, \forall w_u \in \mathcal{W}_u(u)\}$$



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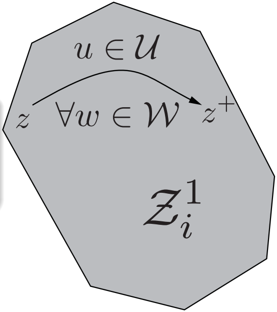
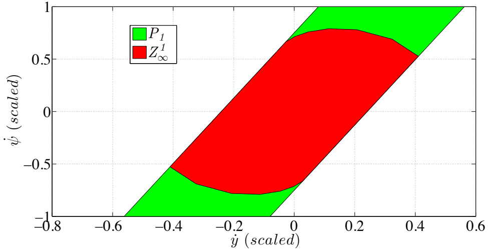
Remark

Largest mode 1 *RCI set* $\mathcal{Z}_\infty^1$ *is the target set in which we want the vehicle state to lie*



RCI Set and the Control Mapping $\mathcal{U}^1_\infty(\cdot)$



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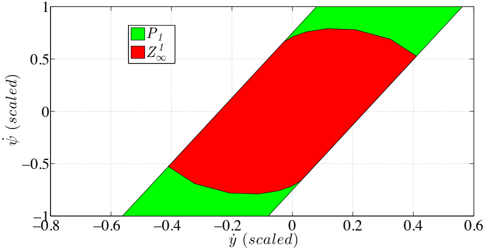

Definition

The control mapping $\mathcal{U}^1_\infty(\cdot)$ corresponding to $\mathcal{Z}^1_\infty$ is defined as:

$$\mathcal{U}^1_\infty(z, \delta_d) := \{u : (z, u, \delta_d) \in \mathcal{Q}_1, z^+ \in \mathcal{Z}^1_\infty, \forall w_u \in \mathcal{W}_u(u)\}$$

- If $z \in \mathcal{Z}^1_\infty$, any choice of input from $\mathcal{U}^1_\infty$ keeps the state in $\mathcal{Z}^1_\infty$

Outside $\mathcal{Z}_{\infty}^{1}$

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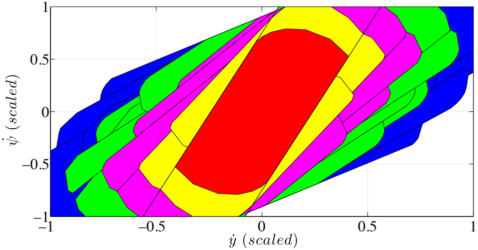
Definition

The N*-step backward reachable sets* $\mathcal{Z}_N$ to the target set $\mathcal{Z}_\infty^1$ are recursively defined as:

$$\mathcal{Z}_k = Pre(\mathcal{Z}_{k-1}), \quad (k = 1, ..., N), \quad \mathcal{Z}_0 = \mathcal{Z}_\infty^1$$

- If the state lies in $\mathcal{Z}_N$, *there exists* a control action that drives the state into $\mathcal{Z}_\infty^1$ in N steps

$\mathcal{Z}_k$ and the Control Mapping $\mathcal{U}_k(\cdot)$



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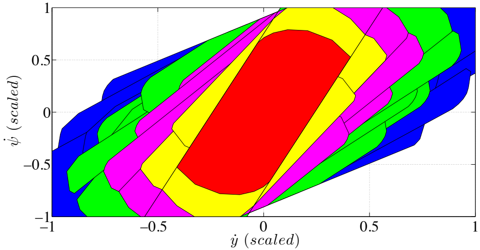

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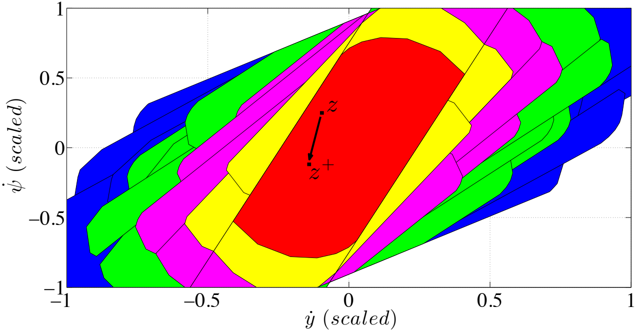
Control Strategy



- If state lies in $\mathcal{Z}^1_\infty$, choose any input from $\mathcal{U}^1_\infty(z, \delta_d)$

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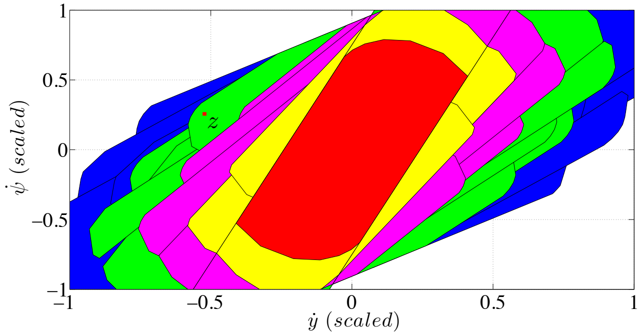

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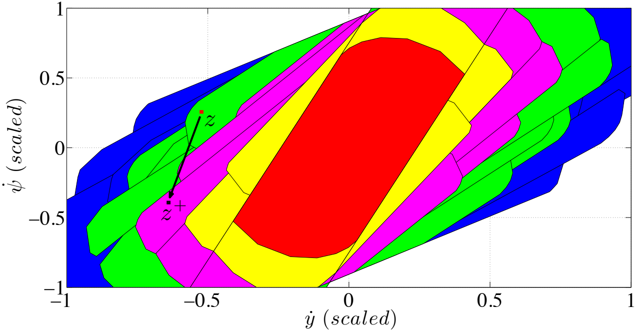

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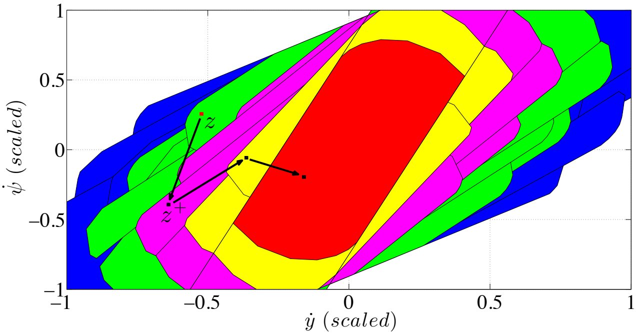

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- Repeating this drives state back into $\mathcal{Z}_\infty^1$

Robust Control with Reference Tracking

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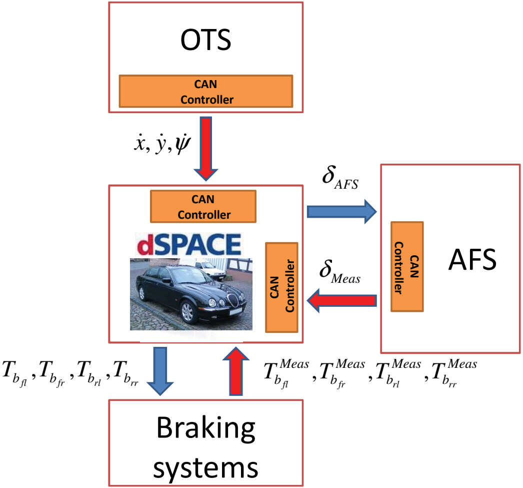
- If the front tires are operating in the saturation zone:

$$u^\star = \text{argmin}_{u \in \mathcal{U}_\star(z,\delta_d)}\,(z^+ - r)^T Q(z^+ - r) + (u - u_{pre})^T R(u - u_{pre}) + P(\alpha_f^+ - \hat{\alpha}_f)^2$$

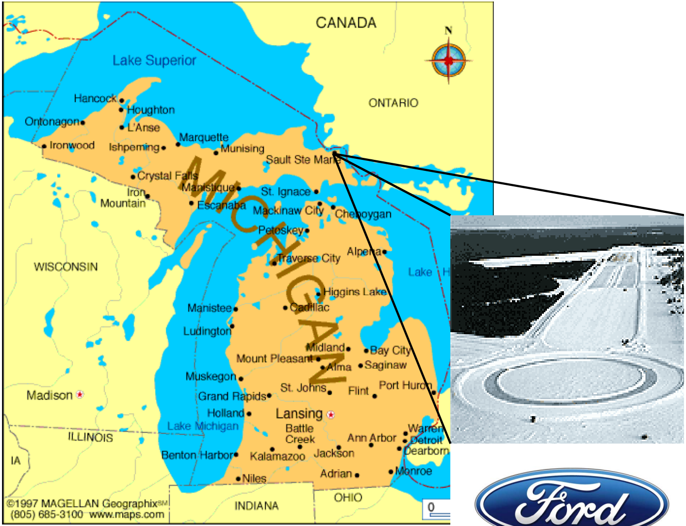
Outline

Experimental Setup

- Jaguar S–type vehicle
- dSPACE rapid prototyping system for real–time computations
- Tests performed at Smithers Winter Test Center, Michigan, USA
- Slippery road surfaces ($\mu \approx 0.3$)
- Sampling time $= 50ms$

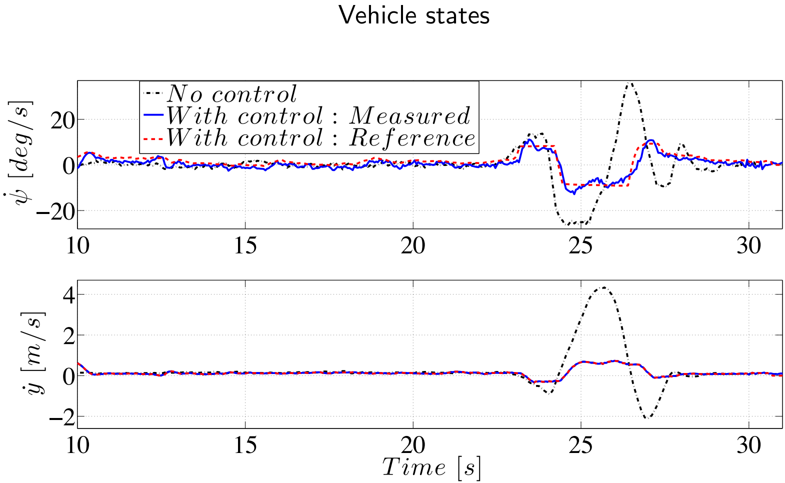


Smithers Winter Test Center, Michigan, USA



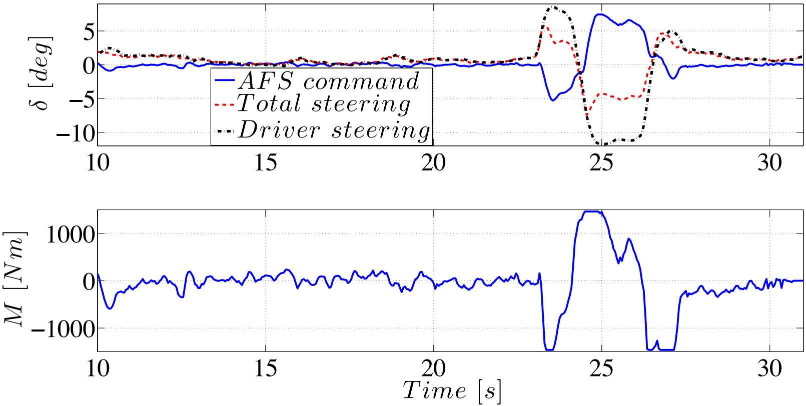
Experiment 1: Double Lane Change

- Double lane change maneuver with an entry speed of 60 kph



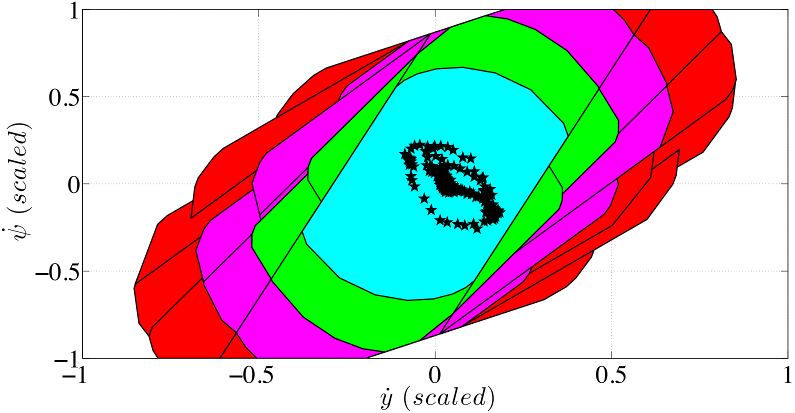
Experiment 1: Double Lane Change

Control inputs



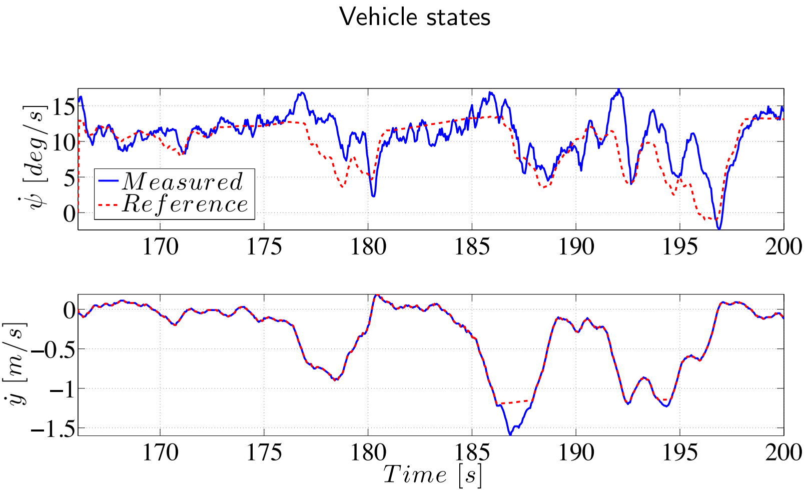
Experiment 1: Double Lane Change

State trajectory superimposed on the backward reachable sets



Experiment 2: Circle on Ice

- Circular maneuver on icy track ($\mu \approx 0.2$) of diameter 110 m at a speed of 40 kph



Experiment 2: Circle on Ice

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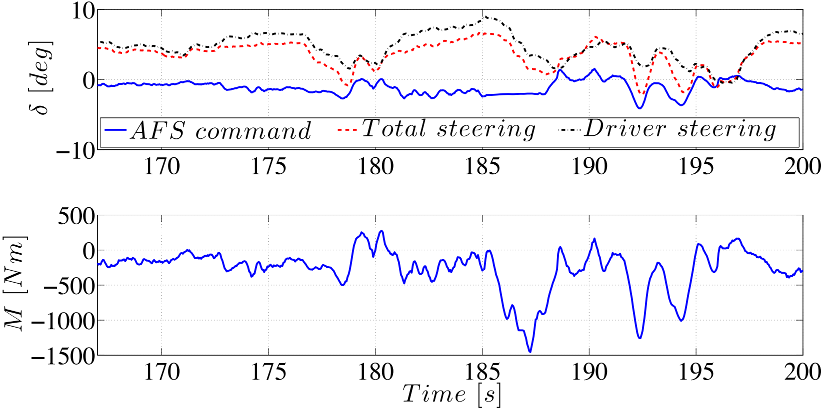

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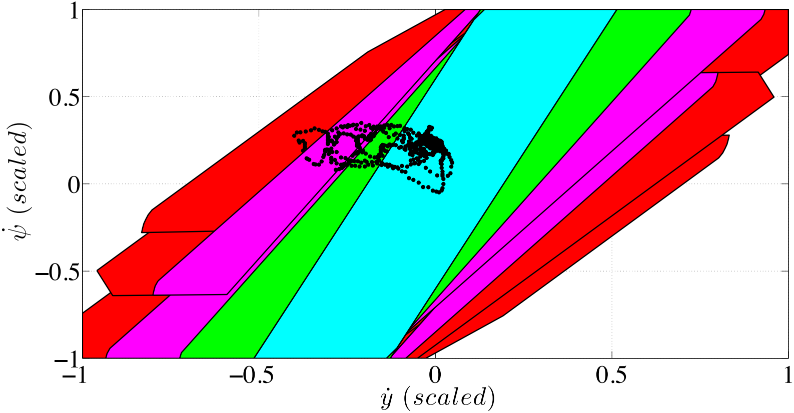

Experimental Results: Video

Robust ESC Experiments

Outline

Conclusions

- Robust vehicle dynamics controller using Active Front Steering (AFS) and differential braking
- Explicitly account for uncertainty in the driver's action in the control design
- Experimental results on low-friction surfaces

Thank You

Any questions?