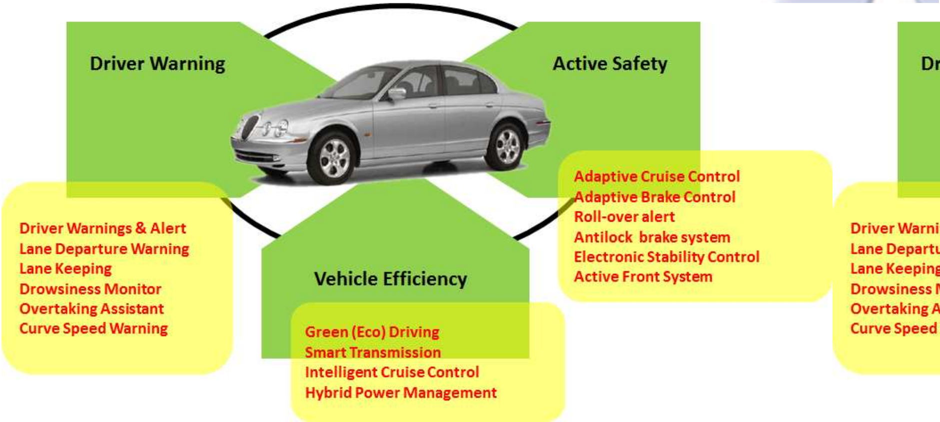
Introduction to Vehicle Dynamics System

Luigi Glielmo, Unisannio

Bertinoro 11 Luglio 2013

Outline

Cybernetic Car



- Driver, vehicle and environment as a closed-loop system, where the driver is the main actuator.

Outline

Longitudinal Dynamics - Mara Tanelli

- The active and safety-oriented control of longitudinal dynamics mainly focuses on traction and braking control systems
- First, an introduction on the considered control problem will be given, illustrating also the tire-road contact forces, which are responsible for generating traction and braking torques
- Secondly, the slip control problem when braking will be addressed, considering: The case of brake-by-wire actuators, which allows a continuous modulation of the braking torque The case of traditional, on-off, hydraulic actuators, which allow only to increase, hold and decrease the braking pressure
- Finally, the case of two-wheeled vehicles will be briefly discussed, and the similarities and differences between traction and braking control will be outlined.

Lateral Dynamics - Luigi Glielmo/Paolo Falcone

- Introduction to vehicle and tire modeling
  - Four Wheel Model
  - Bicycle Model
  - Pacejka's Tire Model
- Model Based Lateral Stability Controller
  - Linear Time Varying - Model Predictive control
  - Invariant Set and Reachability
  - Yaw and lateral dynamics control via braking. An application to a regenerative braking control.
  - Yaw damping via combined front and rear steering

Adaptive Cruise Control and Platooning - Paolo Falcone

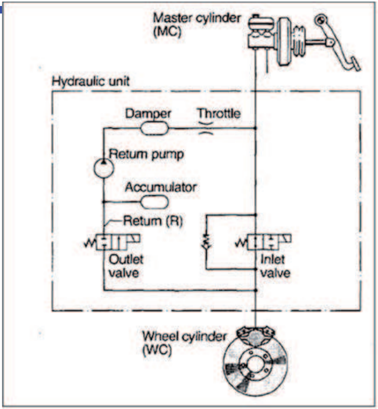
- ACC systems will be first motivated and overviewed, eventually focusing on the underlying control problem formulation. In particular, the problem of designing the longitudinal dynamics control algorithm for an ACC system will be approached in order to guarantee important "string stability" properties for convoys of ACC vehicles.
- The string stability results, provided for a special control structure introduced for ACC systems, will be then generalized for any linear inter-vehicle spacing controller, in the context of decentralized control for homogeneous vehicle platoons. Results will be presented for heterogeneous vehicle platoon as well.
- As time permits, the design of string stable vehicle platoon control algorithms will be formulated in a Model Predictive Control (MPC) framework and the most recent results presented and discussed.

Vertical Dynamics: Suspension Control - Sergio M. Savaresi

- Main elements (spring, damper) of a suspension
- Quarter car: model and dynamic analysis
  - Quarter-car model (single-corner)
  - Quarter-car extended model
  - Linear Model
  - Single Mass Model
  - SkyHook Model
- Electronic suspensions: classification
- Load-leveling and semi-active suspensions
- Semi-active suspensions

Outline

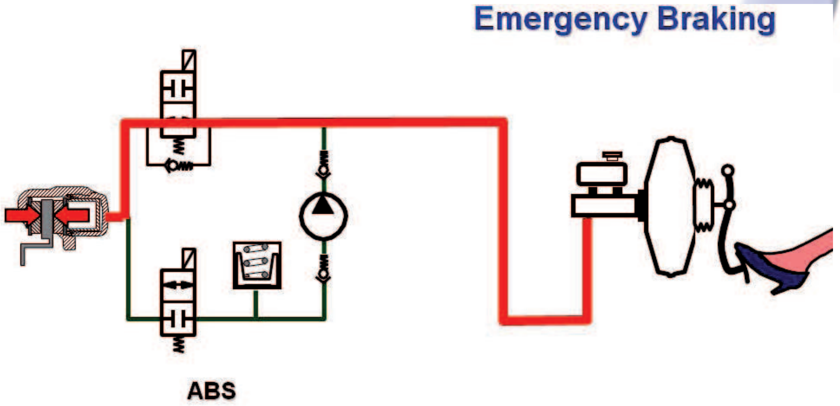
Antilock Braking System



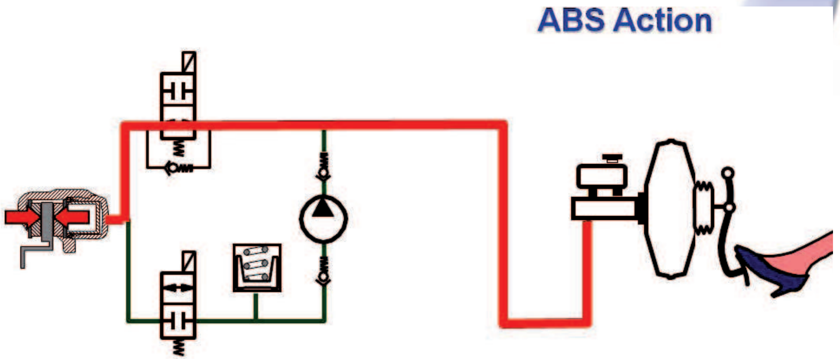
- Avoid wheel locking in braking;
- Guarantee vehicle longitudinal handling in emergency braking;
- Reduce braking space.

- Controlled Variable: Longitudinal Slip
- Control Variable: Brake Pressure to Brake Cylinder
- Based on slip and wheel acceleration, the pressure to BC is:
  - Kept constant;
  - Reduced;
  - Augmented.

Antilock Braking System



Antilock Braking System



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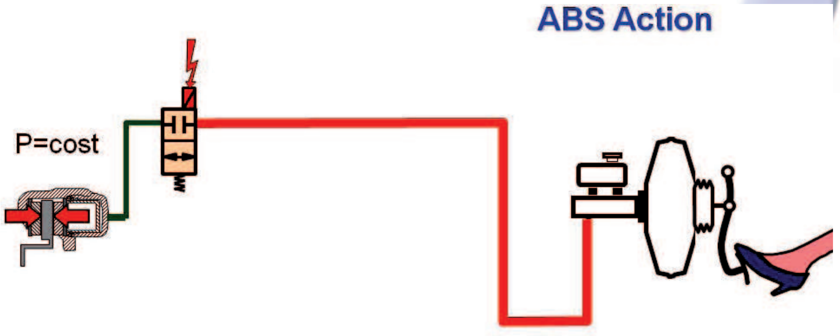
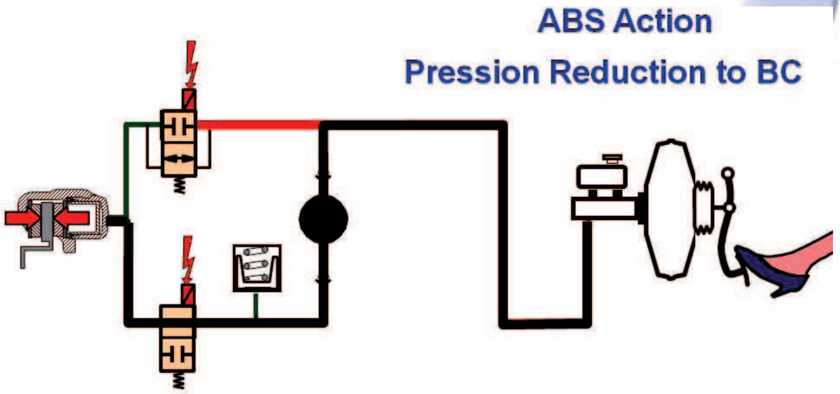
Antilock Braking System



Antilock Braking System



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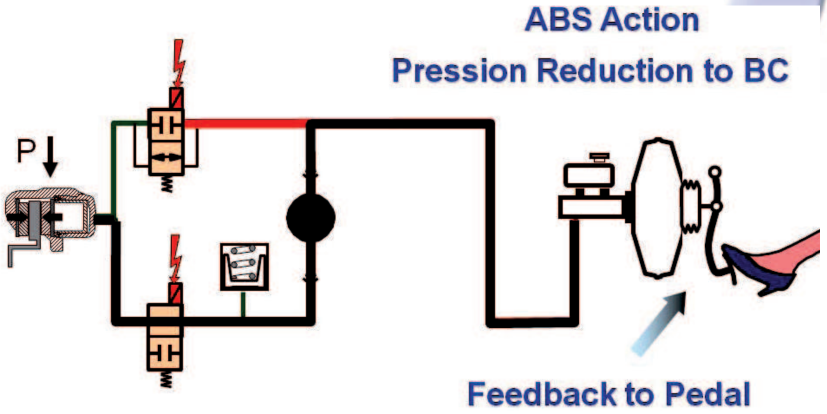
Antilock Braking System



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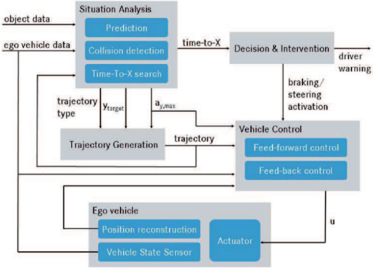
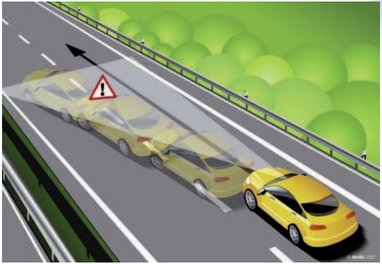
Driver Assistant System



1. Situation analysis predicts how the current driving situation will evolve.
2. This assessment serves to trigger appropriate maneuvers for collision avoidance and collision mitigation.
3. Such maneuvers are realized by specialized vehicle controllers.

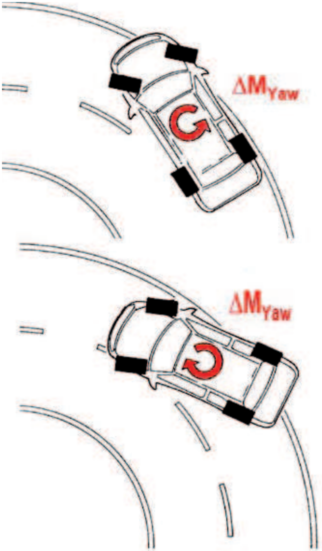
[1] C. G. Keller, T. Dang, H. Fritz, A. Joos, C. Rabe and Dariu M. Gavrila, *Active Pedestrian Safety by Automatic Braking and Evasive Steering*, IEEE Transactions on Intelligent Transportation Systems, vol. 12, no. 4, December 2011.

Lane Keeping



- Lane Keeping Aid uses a camera mounted at the top of the windscreen to monitor the road ahead of the vehicle.
- As the car drifts towards the lane marking, a slight steering torque is automatically applied away from the line, towards the center of the lane.

Electronic Stability Controller



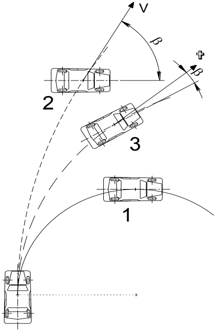
Understeering:

$$|\dot{\psi}| < |\dot{\psi}_{rif}|$$

Oversteering:

$$|\dot{\psi}| > |\dot{\psi}_{rif}|$$

Introduction to ESC



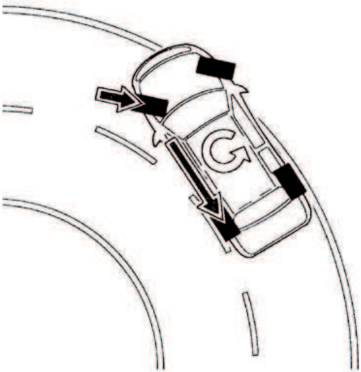
Vehicle on road with high friction coefficient

Vehicle on road with low friction coefficient without VDC

Vehicle on road with low friction coefficient with VDC

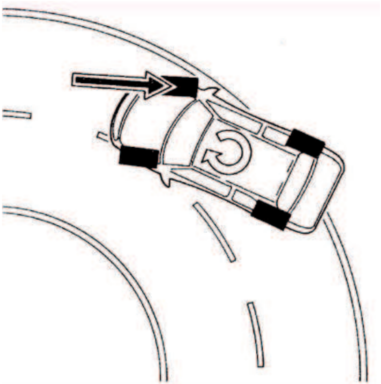
- ESC is an active safety system which control the vehicle handling in emergency manoeuvre.
- From sensors information, the system estimates the vehicle trajectory imposed by the driver.
- ESC command independently the braking pressure on each wheel in order to minimize the difference between desired and real trajectory.

ESC: Understeering action



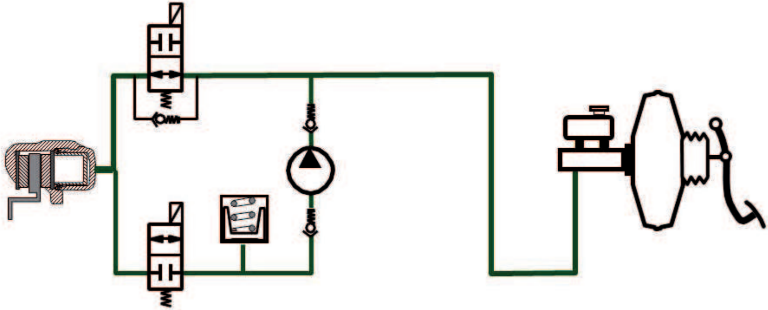
- The controller observes a deviation on frontal axel.
- The controller commands braking on internal wheel (respect to trajectory radius)

ESC: Oversteering action



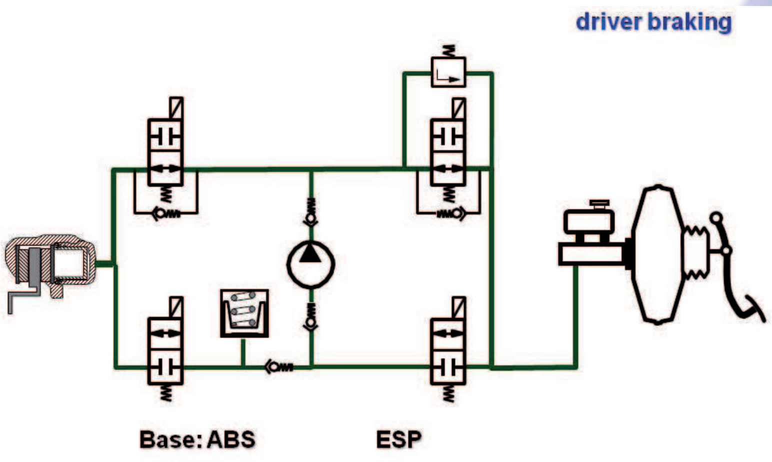
- The controller observes a deviation on rear axel.
- The controller commands braking on external wheels (respect to trajectory radius)

ESC: Hydraulic Behavior



Base: ABS

ESC: Hydraulic Behavior



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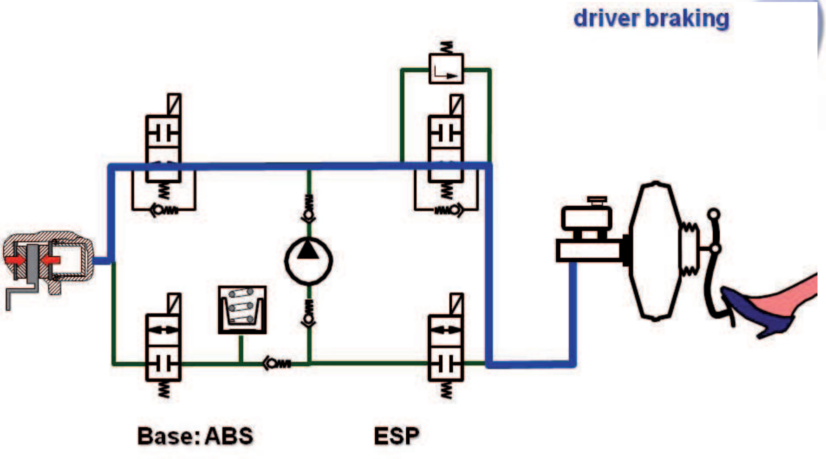
ESC: Hydraulic Behavior



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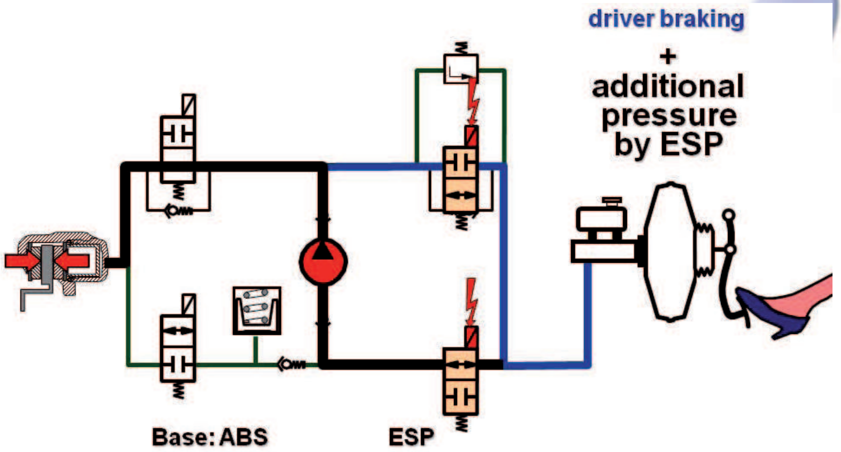
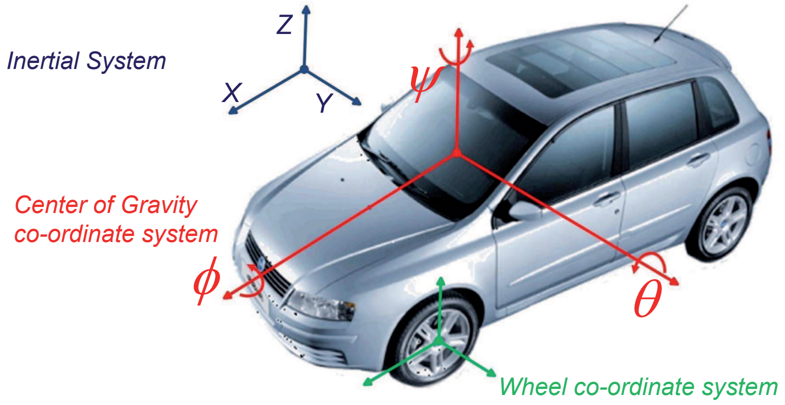
ESC: Hydraulic Behavior



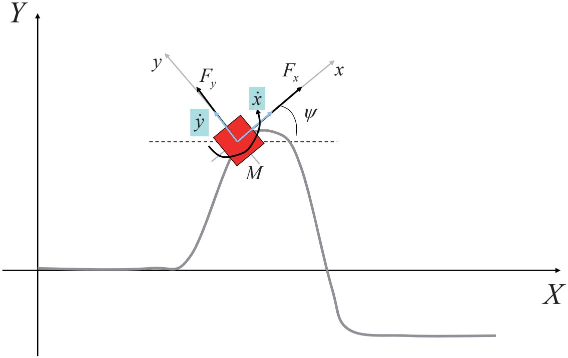
Outline

Coordinate Systems



- With the exception of the fixed inertial system, all co-ordinates move during travel.

Simplified vehicle model

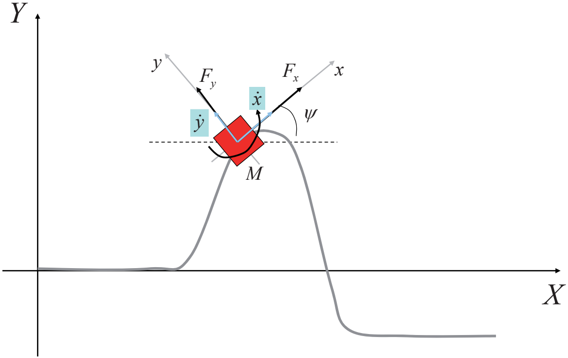


$$m a_x = F_x, \tag{1a}$$

$$m a_y = F_y, \tag{1b}$$

$$I\ddot{\psi} = M. \tag{1c}$$

Simplified vehicle model

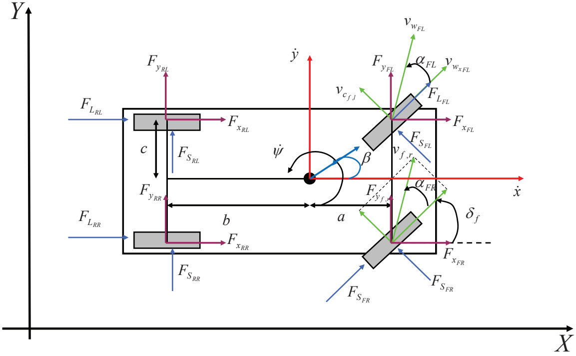


In order to model the planar motion of the vehicle in the inertial frame X-Y the following equations have to be added to (1)

$$\dot{X} = \dot{x}\cos\psi - \dot{y}\sin\psi, \tag{2a}$$

$$\dot{Y} = \dot{x}\sin\psi + \dot{y}\cos\psi, \tag{2b}$$

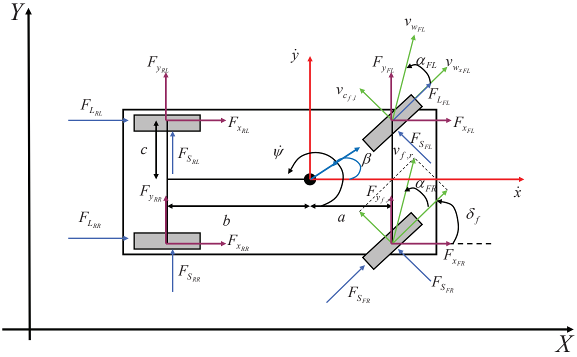
Four wheel model



Remark

The vehicle is a rigid body, i.e., the longitudinal and lateral forces on the right hand sides of (1a) *and* (1b)*, respectively, are computed as the sum of the longitudinal and lateral components* F_x *and* F_y *at the four vehicle wheels.*

Four wheel model

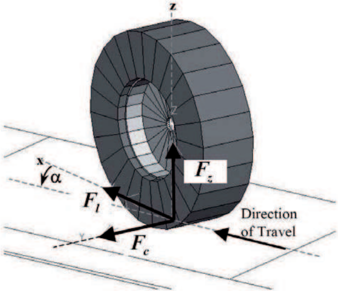


$$m\ddot{x} = m\dot{y}\dot{\psi} + F_{x_{f,l}} + F_{x_{f,r}} + F_{x_{r,l}} + F_{x_{r,r}}, \tag{3a}$$

$$m\ddot{y} = -m\dot{x}\dot{\psi} + F_{y_{f,l}} + F_{y_{f,r}} + F_{y_{r,l}} + F_{y_{r,r}}, \tag{3b}$$

$$I\ddot{\psi} = a\left(F_{y_{f,l}} + F_{y_{f,r}}\right) - b\left(F_{y_{r,l}} + F_{y_{r,r}}\right) + c\left(-F_{x_{f,l}} + F_{x_{f,r}} - F_{x_{r,l}} + F_{x_{r,r}}\right), \tag{3c}$$

Lateral and Longitudinal Tire Forces

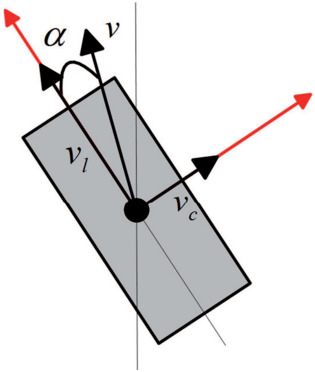


$$F_{x_{\star,\bullet}} = F_{l_{\star,\bullet}} \cos\delta_\star - F_{c_{\star,\bullet}} \sin\delta_\star, \tag{4a}$$
$$F_{y_{\star,\bullet}} = F_{l_{\star,\bullet}} \sin\delta_\star + F_{c_{\star,\bullet}} \cos\delta_\star. \tag{4b}$$

$F_{c_{\star,\bullet}}$ and $F_{l_{\star,\bullet}}$ are nontrivial functions of several parameters. A possible dependency can be described as

$$F_{c_{\star,\bullet}} = f_c(\alpha_{\star,\bullet}, s_{\star,\bullet}, \mu_{\star,\bullet}, F_{z_{\star,\bullet}}), \tag{5a}$$
$$F_{l_{\star,\bullet}} = f_l(\alpha_{\star,\bullet}, s_{\star,\bullet}, \mu_{\star,\bullet}, F_{z_{\star,\bullet}}), \tag{5b}$$

Tire Slip Angle



The slip angle $\alpha_{\star,\bullet}$ represents the angle between the wheel velocity vector $v_{\star,\bullet}$ and the direction of the wheel itself:

$$\alpha_{\star,\bullet} = \arctan \frac{v_{c_{\star,\bullet}}}{v_{l_{\star,\bullet}}}, \tag{6}$$

Tire Slip Angle

$$v_{l_{\star,\bullet}} = v_{x_{\star,\bullet}} \cos\delta_\star + v_{y_{\star,\bullet}} \sin\delta_\star, \tag{7a}$$

$$v_{c_{\star,\bullet}} = -v_{x_{\star,\bullet}} \sin\delta_\star + v_{y_{\star,\bullet}} \cos\delta_\star, \tag{7b}$$

$$v_{x_{f,l}} = \dot{x} - c\dot{\psi}, \quad v_{y_{f,l}} = \dot{y} + a\dot{\psi}, \tag{8a}$$

$$v_{x_{f,r}} = \dot{x} + c\dot{\psi}, \quad v_{y_{f,r}} = \dot{y} + a\dot{\psi}, \tag{8b}$$

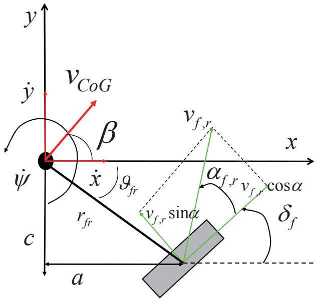
$$v_{x_{r,l}} = \dot{x} - c\dot{\psi}, \quad v_{y_{r,l}} = \dot{y} - b\dot{\psi}, \tag{8c}$$

$$v_{x_{r,r}} = \dot{x} + c\dot{\psi}, \quad v_{y_{r,r}} = \dot{y} - b\dot{\psi}. \tag{8d}$$

Remark

The longitudinal velocities $v_{x_{\star,\bullet}}$ of the left and right wheels have different values during a turn. In a left turn, the yaw rate $\dot{\psi}$ is positive and the velocities of the right wheels are higher than the left wheels ones. In a right turn, the yaw rate is negative and the velocities of the left wheels are higher than the right wheels ones.

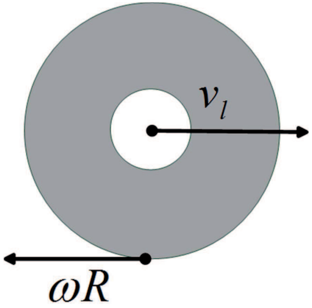
Wheel Velocity Computation



$$v_{x_{f,r}} = v_{CoG} \cos\beta + r_{f,r} \cos\vartheta_{f,r}\dot{\psi}, \tag{9a}$$

$$v_{y_{f,r}} = v_{CoG} \sin\beta + r_{f,r} \sin\vartheta_{f,r}\dot{\psi}. \tag{9b}$$

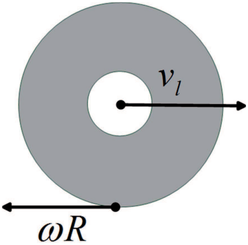
Tire Slip Ratio



The slip ratio $s_{\star,\bullet}$ in (5) is the defined as

$$s_{\star,\bullet} = \begin{cases} \dfrac{r_w \omega_{\star,\bullet}}{v_{l_{\star,\bullet}}} - 1 \text{ if } v_{l_{\star,\bullet}} > r_w \omega_{\star,\bullet},\ v_{l_{\star,\bullet}} \neq 0 \text{ for braking,} \\ 1 - \dfrac{v_{l_{\star,\bullet}}}{r_w \omega_{\star,\bullet}} \text{ if } v_{l_{\star,\bullet}} < r_w \omega_{\star,\bullet},\ \omega_{\star,\bullet} \neq 0 \text{ for driving.} \end{cases} \tag{10}$$

Angular speeds



The wheel angular speeds $\omega_{\star,\bullet}$ in (10) are obtained by integrating the following set of differential equations:

$$J_{w,\star}\dot{\omega}_{\star,\bullet} = -F_{l_{\star,\bullet}} r_w - T_{b_{\star,\bullet}} + T_{t_{\star,\bullet}} - b \cdot \omega_{\star,\bullet}, \tag{11}$$

$$T_{t_{f,l}} + T_{t_{f,r}} + T_{t_{r,l}} + T_{t_{r,r}} \leq T_{\text{eng}}. \tag{12}$$

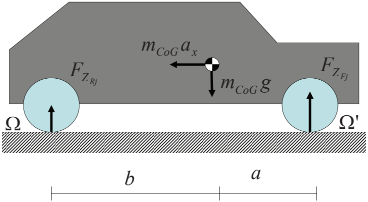
Normal Forces: Longitudinal Load Transfer

- By disregarding suspension dynamics, the quarter vehicle forces are identical to the wheel ground contact force $F_{Z_\star}$.
- The force due to longitudinal acceleration (ma_x) at the CoG causes a pitch torque which reduces the front axle load and increases the rear axle load.
- Constructing the torque balance at the rear axis contact point yields:

$$(a+b)F_{z_f} = bmg - hma_x. \tag{13}$$

Thus,

$$F_{z_f} = \frac{bmg}{a+b} - \frac{hma_x}{a+b}. \tag{14}$$



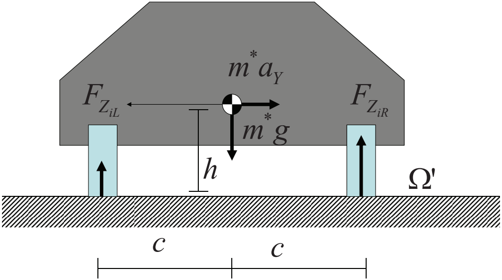
Normal Forces: Lateral Load Transfer

- During cornering the lateral acceleration causes a roll torque
- The two axles are considered to be decoupled from one another. In the case of front axle a virtual mass m^* is used:

$$m^* = \frac{F_{z_f}}{g}. \tag{15}$$

- From the torque balance equation at the ground contact point of the front left wheel:

$$F_{z_{f,r}} 2c = F_{z_f} c + m^* a_Y h. \tag{16}$$



Normal Forces: Total Load Transfer

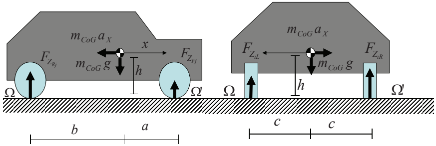
Substituting the virtual mass (15) and F_{z_f} from (14) and solving for $F_{z_{f,r}}$, it is possible compute the front right dynamic wheel force. By analogy the wheel forces for the other three wheels can be derived:

$$F_{z_{f,l}} = mg\frac{b}{2(a+b)} - ma_X\frac{h}{2(a+b)} - ma_Y\frac{h}{2c}, \tag{17a}$$

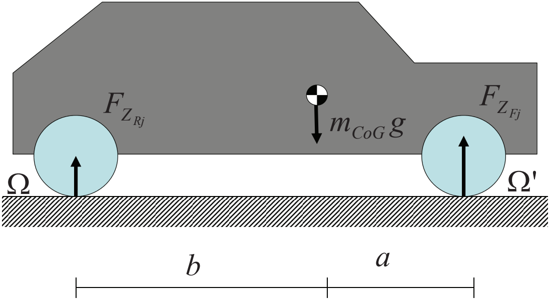
$$F_{z_{f,r}} = mg\frac{b}{2(a+b)} - ma_X\frac{h}{2(a+b)} + ma_Y\frac{h}{2c}, \tag{17b}$$

$$F_{z_{r,l}} = mg\frac{a}{2(a+b)} + ma_X\frac{h}{2(a+b)} - ma_Y\frac{h}{2c}, \tag{17c}$$

$$F_{z_{r,r}} = mg\frac{a}{2(a+b)} + ma_X\frac{h}{2(a+b)} + ma_Y\frac{h}{2c}. \tag{17d}$$



Normal Force: Static Assumption



Remark

The normal forces $F_{z_{\star,\bullet}}$ are constant and distributed between the front and rear axles based on the geometry of the car model (described by the parameters a and b):

$$F_{z_{f,\bullet}} = \frac{bmg}{2(a+b)}, \quad F_{z_{r,\bullet}} = \frac{amg}{2(a+b)}. \tag{18}$$

Complete four wheel model

Using the equations (2), (3)–(18) the nonlinear vehicle dynamics can be described by a differential equation as

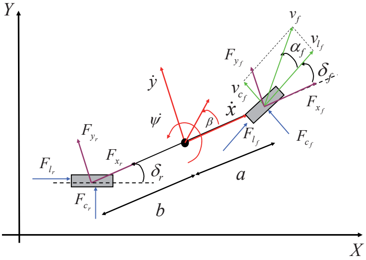
$$\dot{\xi}(t) = f^{4w}_{\mu(t)}(\xi(t), u(t)), \tag{19}$$

where the state and input vectors are
$\xi = [\dot{x}, \dot{y}, \psi, \dot{\psi}, X, Y, \omega_{f,l}, \omega_{f,r}, \omega_{r,l},\ \omega_{r,r}]$ and
$u = [\delta_f,\ T_{b_{f,l}},\ T_{b_{f,r}},\ T_{b_{r,l}},\ T_{b_{r,r}},\ T_{t_{f,l}},\ T_{t_{f,r}},\ T_{t_{r,l}},\ T_{t_{r,r}}]$, respectively,
and $\mu(t) = [\mu_{f,l}(t),\ \mu_{f,r}(t),\ \mu_{r,l}(t),\ \mu_{r,r}(t)]$.

The Bicycle Model

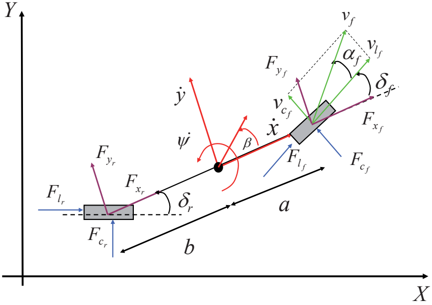
Remark

At front and rear axles, the left and right wheels are lumped in a single wheel.



The states of the bicycle model are the lateral and longitudinal velocities in the body frame, the yaw angle, the yaw rate, the lateral and longitudinal vehicle coordinates in an inertial frame. The inputs are the front steering angle and the forces at the wheel.

The Bicycle Model: equation



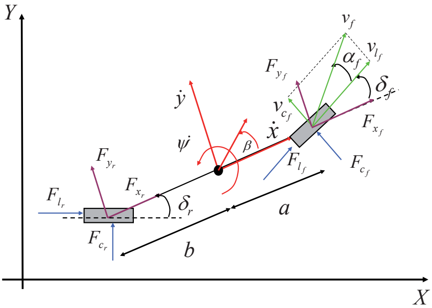
The equations (1) can be rewritten as follows:

$$m\ddot{x} = m\dot{y}\dot{\psi} + 2F_{x_f} + 2F_{x_r}, \tag{20a}$$

$$m\ddot{y} = -m\dot{x}\dot{\psi} + 2F_{y_f} + 2F_{y_r}, \tag{20b}$$

$$I\ddot{\psi} = 2aF_{y_f} - 2bF_{y_r}. \tag{20c}$$

The Bicycle Model: Forces

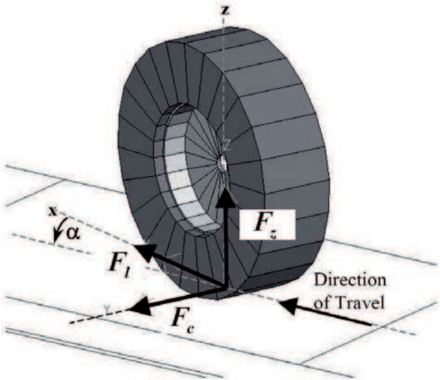


The longitudinal and lateral forces $F_{x_\star}$ and $F_{y_\star}$ in (20) are computed from the longitudinal and cornering tire forces $F_{l_\star}$ and $F_{c_\star}$ as

$$F_{x_\star} = F_{l_\star} \cos\delta_\star - F_{c_\star} \sin\delta_\star, \tag{21a}$$

$$F_{y_\star} = F_{l_\star} \sin\delta_\star + F_{c_\star} \cos\delta_\star. \tag{21b}$$

The Bicycle Model: Forces



For the bicycle model (20), the tire forces can be rewritten as follows

$$F_{l_\star} = f_l(\alpha_\star, s_\star, \mu_\star, F_{z_\star}), \tag{22a}$$

$$F_{c_\star} = f_c(\alpha_\star, s_\star, \mu_\star, F_{z_\star}). \tag{22b}$$

The Bicycle Model: Tire Slip Angle

The tire slip angle in (22) is computed through the following equation

$$\alpha_\star = \arctan \frac{v_{c_\star}}{v_{l_\star}}; \tag{23}$$

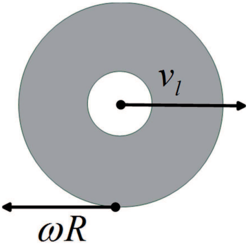
the longitudinal and cornering tire velocities $v_{l_\star}$ and $v_{c_\star}$ are

$$v_{l_\star} = v_{x_\star} \cos \delta_\star + v_{y_\star} \sin \delta_\star, \tag{24a}$$
$$v_{c_\star} = -v_{x_\star} \sin \delta_\star + v_{y_\star} \cos \delta_\star; \tag{24b}$$

where the longitudinal and lateral tire velocities in the body frame $v_{x_\star}$ and $v_{y_\star}$ are computed from the vehicle states as

$$v_{x_f} = \dot{x} \qquad v_{x_r} = \dot{x}, \tag{25a}$$
$$v_{y_f} = \dot{y} + a\dot{\psi} \qquad v_{y_r} = \dot{y} - b\dot{\psi}. \tag{25b}$$

The Bicycle Model: Tire slip ratio

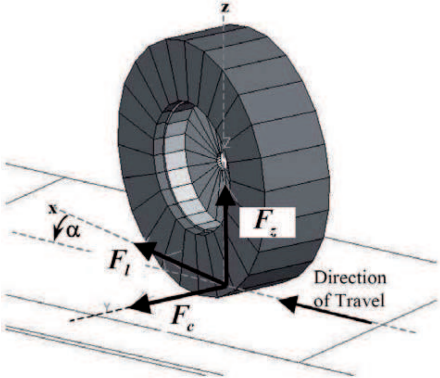


The slip ratio $s_\star$ in (22) is the defined as

$$s_\star = \begin{cases} \dfrac{r_w \omega_\star}{v_{l_\star}} - 1 \text{ if } v_{l_\star} > r_w \omega_\star,\ v_{l_\star} \neq 0 \text{ for braking} \\ 1 - \dfrac{v_{l_\star}}{r_w \omega_\star} \text{ if } v_{l_\star} < r_w \omega_\star,\ \omega_\star \neq 0 \text{ for driving,} \end{cases} \tag{26}$$

where $\omega_\star$ can be computed as average of the left and right wheels angular velocity for $\star \in \{f, r\}$.

The Bicycle Model: Tire normal forces



$$F_{z_f} = \frac{bmg}{2(a+b)}, \quad F_{z_r} = \frac{amg}{2(a+b)}. \tag{27}$$

They are the same of (18), where the second subscript, indicating the left and right sides, has to be removed.

The Bicycle Model: Final Model

Remark

The friction coefficient μ and the slip ratio s are assumed to be equal at the left and right wheels, i.e., no μ-split occurs, and same braking and acceleration take place at the left and right sides.

The nonlinear vehicle dynamics described by the equations (2), (18) and (20)–(26) can be expressed as

$$\dot{\xi}(t) = f^{2w}_{s(t),\mu(t)}(\xi(t), u(t)) \tag{28}$$

where $\mu(t) = [\mu_f(t),\ \mu_r(t)]$ and $s(t) = [s_f(t),\ s_r(t)]$. The state and input vectors are $\xi = [\dot{x},\ \dot{y},\ \psi,\ \dot{\psi},\ X,\ Y]$ and $u = \delta_f$, respectively. In the following δ_r is assumed to be zero at any time instant.

Remark

For control design purposes, the slip ratio s and friction coefficient μ in (28) can be considered as known external disturbances.

The Linear Bicycle Model

A linear bicycle model can be cast by using small angles approximation and linearizing the tire lateral forces which yields

$$\begin{bmatrix} \dot{\beta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} -\frac{c_F+c_R}{mv} & \frac{c_R b - c_F a}{mv^2} - 1 \\ \frac{c_R b - c_F a}{mab} & -\frac{c_F a^2 + c_R b^2}{mvab} \end{bmatrix} \begin{bmatrix} \beta \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} \frac{c_F}{mv} \\ \frac{c_F}{mb} \end{bmatrix} \delta_f, \qquad (29)$$

where

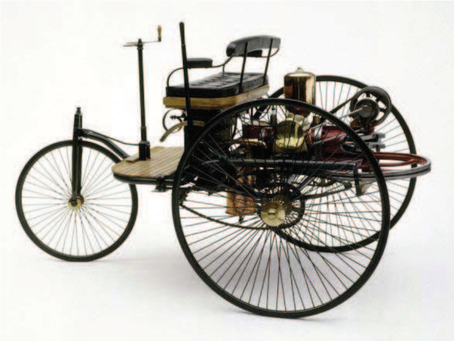
$$c_F = \frac{\partial}{\partial \alpha_f} F_{cf}(\alpha_f, s_f, \mu, F_z)\Big|_{\substack{\alpha_f=0 \\ s_f=0}} \quad c_R = \frac{\partial}{\partial \alpha_r} F_{cr}(\alpha_r, s_r, \mu, F_z)\Big|_{\substack{\alpha_r=0 \\ s_r=0}}$$

- Notice that the bicycle model is always stable provided $c_F a < c_R b$ (i.e., the vehicle is inherently understeering) otherwise, for an oversteering vehicle, it is required

$$v < v_{ch} = \sqrt{\frac{c_F c_R l^2}{m(c_F a - c_R b)}}$$

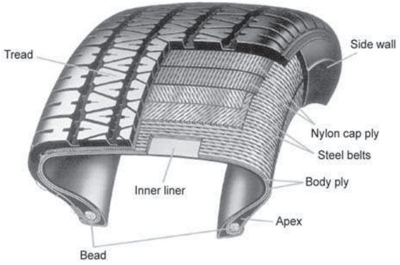
Outline

Tire Role



There are not (yet?) alternative materials or technologies that could substitute tires on the car.
Their main characteristic is deformability that, together with lightless, are able to keep the interaction tire-road even when on the road there are little asperities.
The material composition allows the vehicle to have a good adherence so as to obtain good dynamics performance.

Modeling State of Art



To model a tyre and the interaction with the environment different modeling approaches can be used:

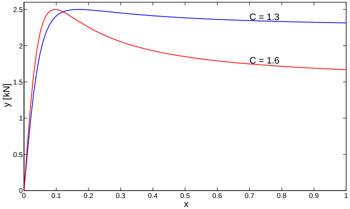
- analytical models,
- empirical models,
- physical models.

Most of the existing tire models are predominantly "semi-empirical" in nature. That is, the tire model structure is determined through analytical considerations, and key parameters depend on tire data measurements.

Pacejka's Tire Model

- This is a complex, semi-empirical model in which the longitudinal and cornering forces depend by the normal force, slip angle, surface friction coefficient, and longitudinal slip.
- It describes the tire dynamics over operating ranges of slip ratio and tire slip angle, including both the linear and nonlinear regions.
- The tire forces saturation occurring in the nonlinear region are described by Pacejka's "magic" formula:

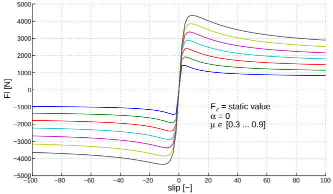
$$y(x) = D \sin\left(C \arctan\left(Bx - E\left[Bx - \arctan(Bx)\right]\right)\right) \tag{30}$$

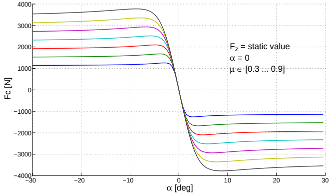


Pacejka's Tire Model: Assumption

Remark

The model (30) can only describe the longitudinal and tire force in pure braking/driving (tire side slip angle $\alpha = 0$) and pure cornering (slip ratio $s = 0$). In order to model combined braking/driving and cornering, equation (30) has to be modified with corrective element.

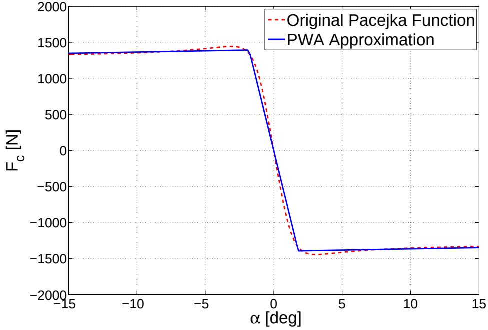




PWA Approximation of Lateral Force

For control design it is convenient to have either a Linear or a PieceWise Affine Approximation of pure braking/driving tire force:

$$F_{c_{\star,\bullet}} = \begin{cases} C_{\alpha_{\text{sat}}}\alpha_{\star,\bullet} - C_{s_{\text{off}}} & \text{if } \alpha_{\star,\bullet} < -\alpha^{\star}_{\star,\bullet} \\ C_{\alpha_{\text{sat}}}\alpha_{\star,\bullet} & \text{if } -\alpha^{\star}_{\star,\bullet} < \alpha_{\star,\bullet} < \alpha^{\star}_{\star,\bullet} \\ C_{\alpha_{\text{sat}}}\alpha_{\star,\bullet} + C_{s_{\text{off}}} & \text{if } \alpha_{\star,\bullet} > \alpha^{\star}_{\star,\bullet} \end{cases} \tag{31}$$



Outline

References

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