Vehicle Lateral Dynamics Control

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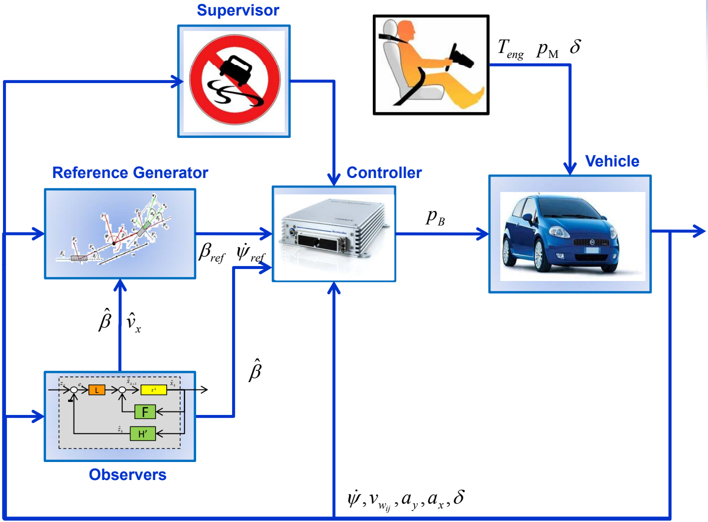
Outline

1. General Control Scheme
2. Differential Braking Model Predictive Control
3. Simulation results

General Control Scheme



Outline

1. General Control Scheme

2. Differential Braking Model Predictive Control
   - Control Model Design
   - LTV–MPC Problem Formulation
   - Bounds on the control inputs
   - A reduced model for slip control
   - A slip control strategy
   - Feedback action of slip controller

3. Simulation results

Control Model Design

The model presented in the first part (two states model) is nonlinear. To reduce the computational load, we first compute the linearization around the current state (β, $\dot{\psi}$).

Further, in our control model we consider as input the variation ΔF_L of the tire braking (i.e., negative) forces constrained to

$$-F_L^{\max} \leq F_L + \Delta F_L \leq 0 \tag{1}$$

so that

$$\Delta F_L^{\min} \leq \Delta F_L \leq \Delta F_L^{\max}, \tag{2a}$$

$$\Delta \dot{F}_L^{\min} \leq \Delta \dot{F}_L \leq \Delta \dot{F}_L^{\max}, \tag{2b}$$

where $\Delta F_L^{\min} := -F_L^{\max} - F_L$ and $\Delta F_L^{\max} := -F_L$. Inequalities (2a) and (2b) represent physical bounds: (2a) describes the tire forces domain in a particular working point, and (2b) represents the slew-rate of the braking system.

LTV–MPC Problem Formulation

The control goal is to maintain the errors $e_{\widetilde{\beta}}$ and $e_{\dot{\psi}}$ close to zero. The linearized model around the working point $(v, \widetilde{\beta}, \delta, F_L)$ is

$$\dot{e} = A\left(v, \widetilde{\beta}, \delta, F_L\right) e + B\left(v, \widetilde{\beta}, \delta\right) \begin{bmatrix} \Delta F_{L_{FL}} \\ \Delta F_{L_{FR}} \\ \Delta F_{L_{RL}} \\ \Delta F_{L_{RR}} \end{bmatrix} + d\left(v, \widetilde{\beta}, \dot{\psi}, \delta, F_{L_{ij}}, F_{S_{ij}}\right), \tag{3}$$

where $e = \left[e_{\widetilde{\beta}}, e_{\dot{\psi}}\right]^T$ and $A\left(v, \widetilde{\beta}, \delta, F_L\right) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ with

LTV–MPC Problem Formulation

$$a_{11} = -\frac{c_F}{mv}\left(\cos\delta + \delta\sin\delta\right) - \frac{c_R}{m} - \frac{1}{mv}\left(F_{L_{FR}} + F_{L_{FL}}\right)\cos\delta + \tag{4a}$$

$$-\frac{1}{mv}\left(F_{L_{RR}} + F_{L_{RL}}\right) - 2\frac{c_F}{mv}\widetilde{\beta}\sin\delta - \frac{c_F}{mv^2}l_a\dot{psi}\sin\delta, \tag{4b}$$

$$a_{12} = -\frac{l_a c_F}{mv^2}\left(\cos\delta + \widetilde{\beta}\sin\widetilde{\beta}\right) + \frac{l_b c_R}{mv^2} - 1, \tag{4c}$$

$$a_{21} = -\frac{l_a c_F}{J_z}\cos\delta + \frac{l_b c_R}{J_z} + \left(c_{FR}\sin\delta - c_{FL}\sin\delta\right)l_c, \tag{4d}$$

$$a_{22} = -\frac{l_a^2 c_F}{J_z v}\cos\delta - \frac{l_b^2 c_R}{J_z v} + \left(c_{FR}\sin\delta - c_{FL}\sin\delta\right)l_c; \tag{4e}$$

and

LTV–MPC Problem Formulation

$$B\left(v, \widetilde{\beta}, \delta\right) = \begin{bmatrix} \frac{\sin\delta - \widetilde{\beta}\cos\delta}{mv} & \frac{\sin\delta - \widetilde{\beta}\cos\delta}{mv} & -\frac{\widetilde{\beta}}{mv} & -\frac{\widetilde{\beta}}{mv} \\ \frac{l_a\sin\delta - l_c\cos\delta}{J_z} & \frac{l_a\sin\delta + l_c\cos\delta}{J_z} & \frac{l_c}{J_z} & -\frac{l_c}{J_z} \end{bmatrix},$$

$$d\left(v, \widetilde{\beta}, \dot{\psi}, \delta, F_{L_{ij}}, F_{S_{ij}}\right) = \begin{bmatrix} f_1\left(v, \widetilde{\beta}, \dot{\psi}, \delta, F_{L_{ij}}, F_{S_{ij}}\right) \\ f_2\left(v, \widetilde{\beta}, \dot{\psi}, \delta, F_{L_{ij}}, F_{S_{ij}}\right) \end{bmatrix}.$$

The model and its constraints are then discretized by using backward Euler's method with sampling time T_s, thus obtaining

$$x_{k+1} = A_T(k)x_k + B_T(k)u_k + d_T(k) \tag{5}$$

where $u_k = \begin{bmatrix} \Delta F_{L_{FL}} & \Delta F_{L_{FR}} & \Delta F_{L_{RL}} & \Delta F_{L_{RR}} \end{bmatrix}' |_{t=kT_s}$ is the control input, $A_T(k) = I + A\left(v, \widetilde{\beta}, \delta, F_L\right)|_{t=kT_s} T_s$ is the system matrix, $B_T(k) = B\left(v, \widetilde{\beta}, \delta\right)|_{t=kT_s} T_s$ is the input matrix, $d_T(k) = d\left(v, \widetilde{\beta}, \dot{\psi}, \delta, F_{L_{ij}}, F_{S_{ij}}\right)|_{t=kT_s} T_s$.

LTV–MPC Problem Formulation

We now consider a prediction horizon H_pT_s, for some integer H_p, in which the following approximations hold:

$$\overline{A}_T = A_T(k) \cong A_T(k+1) \cong \ldots \cong A_T(k+H_p-1), \tag{6a}$$

$$\overline{B}_T = B_T(k) \cong B_T(k+1) \cong \ldots \cong B_T(k+H_p-1), \tag{6b}$$

$$\overline{d}_T = d_T(k) \cong d_T(k+1) \cong \ldots \cong d_T(k+H_p-1). \tag{6c}$$

We also consider a control horizon H_cT_s, for some integer $H_c \leq H_p$, and define $\mathcal{U}_k = \begin{bmatrix} u'_k & \ldots & u'_{k+H_c-1} \end{bmatrix}'$.

LTV–MPC Problem Formulation

At each time k we solve the following optimization problem:

$$\mathcal{U}_k^* = \arg\min_{\mathcal{U}_k} \sum_{h=0}^{H_p-1} \left(x_{k+h+1}^T \, Q \, x_{k+h+1} + u_{k+h}^T \, R \, u_{k+h} \right) \tag{7a}$$

subject to

$$x_{k+h+1} = \overline{A}_T x_{k+h} + \overline{B}_T u_{k+h} + \overline{d}_T, \tag{7b}$$

$$x_k = \begin{bmatrix} e_\beta(kT_s) \\ e_{\dot\psi}(kT_s) \end{bmatrix}, \tag{7c}$$

$$u_{k+h+1} = u_{k+h} \qquad \forall h \geq H_c - 1, \tag{7d}$$

$$u^{\min} \leq u_{k+h} \leq u^{\max}, \tag{7e}$$

$$\Delta u^{\min} \leq u_{k+h} - u_{k+h-1} \leq \Delta u^{\max}, \tag{7f}$$

where $u_{-1} = 0$. Inequality (7e) is related to ((2a)) and inequality (7f) is related to (2b).

LTV–MPC Problem Formulation

Even though the MPC algorithm is developed and implemented in discrete time, it is notationally convenient to employ a continuous time description in the remaining part of the work. To do so we denote:

$$u(t)_{\mathrm{MPC}} = [\mathrm{hold}(t; \Delta F_k, T_s))/T_s \tag{8}$$

where, with some abuse of notation,

$$\mathrm{hold}(t; u_k, T_s)) = u_k \qquad \text{for } t \in [kT_s, (k+1)T_s). \tag{9}$$

LTV–MPC Problem Formulation

Remark

Since the cost function (7a) *is quadratic and the constraints* (7b)–(7f) *are linear, the optimization problem* (7) *is convex and can be solved with an efficient quadratic programming (QP) solver.*

Remark

We denote by $\mathcal{U}_k^* = [u_k^*, \dots, u_{k+H_c-1}^*]'$ *the sequence of optimal braking torques computed at time* k *by solving problem (7) from the current observed state* x_k*. Then the first element* u_k^* *of* $\mathcal{U}_k^*$ *is actually applied to the system at time* k*.*

Bounds on the control inputs

If we represent the longitudinal and lateral forces that the road can exert on the tire as showed in the next figure, they belong to an area depending on the conditions of road μ and the vertical force F_z. We assume (but this can be actually justified on the basis of the combined Pacejka's formulas) that the area can be approximated by the ellipsoid.

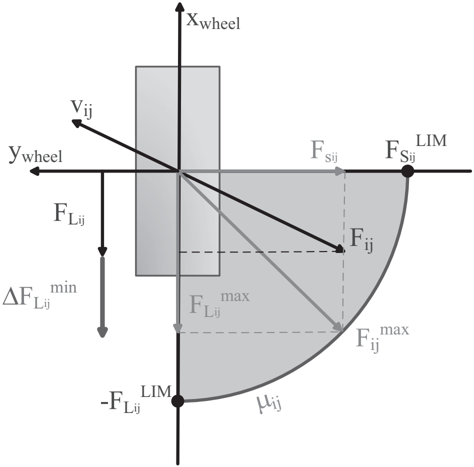
$$\varepsilon F_L^2 + F_S^2 = r^2, \tag{10}$$

where $r := F_S^{LIM}$ and $\varepsilon := \left(F_S^{LIM}/F_L^{LIM}\right)^2$; F_L^{LIM} and F_S^{LIM} represent the maximum longitudinal force and lateral force respectively, defined as

$$F_L^{LIM}\left(\mu, F_z\right) := \max_s f_L\left(s; \mu, F_z\right), \tag{11a}$$

$$F_S^{LIM}\left(\mu, F_z\right) := \max_\alpha f_S\left(\alpha; \mu, F_z\right). \tag{11b}$$

Bounds on the control inputs



Bounds on the control inputs

Given the actual side force F_S, the maximum achievable longitudinal force that avoids the unstable slipping of the wheel is given by

$$F_L^{\max} := \sqrt{\frac{1}{\varepsilon}\left[r^2 - (F_S)^2\right]}. \tag{12}$$

Finally we point out that, since we are considering the braking action of the tires, rather than the acceleration action, the longitudinal forces F_L will be negative (see figure) and the "maximum" braking forces corresponding to a side force F_S will be $-F_L^{\max}$.

A reduced model for slip control

- The desired variation of longitudinal forces is achieved by the braking system.
- The slip controller computes the desired longitudinal slip corresponding to the desired longitudinal force and computes the braking force that has to be applied.
- For control design we will consider the equation

$$f\left(F_{L_{ij}}, T_{\mathrm{eng}_{ij}}, T_{B_{ij}}\right) = \frac{1}{J_w}\left(-rF_{L_{ij}} - T_{B_{ij}} + T_{\mathrm{eng}_{ij}}\right). \qquad (13)$$

- thanks to the fast time response of the wheel, we can neglect the other dynamics by considering as constant the yaw rate, vehicle side sleep angle and the vehicle velocity during the time interval of slip control intervention.

A reduced model for slip control

With this assumption the model of angular wheel velocity becomes

$$J_w \dot{\omega} = -r f_L(s) + T_{\text{eng}} - T_B, \tag{14}$$

and the derivative of the slip ratio can be written as

$$\dot{s} = \frac{r\dot{\omega}v}{\max\{v^2, (r\omega)^2\}}. \tag{15}$$

We distinguish two cases:

Braking When $v \geq r\omega$, then $s \in [-1, 0]$ and Equation (15) yields

$$\dot{s} = \frac{r\dot{\omega}}{v}, \tag{16}$$

so that Equation (14) becomes

$$\dot{s} = \left(\frac{r}{vJ_w}\right)\left(-r f_L(s) + T_{\text{eng}} - T_B\right). \tag{17}$$

A reduced model for slip control

Traction When $r\omega \geq v$ then $s \in [0, 1]$ and Equation (15) can be written as

$$\dot{s} = \frac{r\dot{\omega}v}{r^2\omega^2} = \frac{r\dot{\omega}}{v}(1-s)^2 \tag{18}$$

so that (14) becomes

$$\dot{s} = \left(\frac{r}{vJ_w}\right)\left(-rf_L(s) + T_{\text{eng}} - T_B\right)(1-s)^2. \tag{19}$$

Notice that the roots of

$$rf_L(\overline{s}) = T_{\text{eng}} - T_B \tag{20}$$

are the equilibrium points $\bar{s}$ of both (17) and (19) with $s \in [-1, 0]$ and $s \in [0, 1]$, respectively. Such equilibrium points may or may not exist (value of the RHS of equation (20)), and there may be one or two equilibrium points. If $\partial f_L(\bar{s}) / \partial s$ is positive, $\overline{s}$ is asymptotically stable so that we define the *stable interval* as the interval of s where the above derivative is positive.

A reduced model for slip control

One of the goals of vehicle dynamic control is to keep the longitudinal slip s inside the stable interval where typically the longitudinal slip is close to zero and $(1-s)^2 \cong 1$.

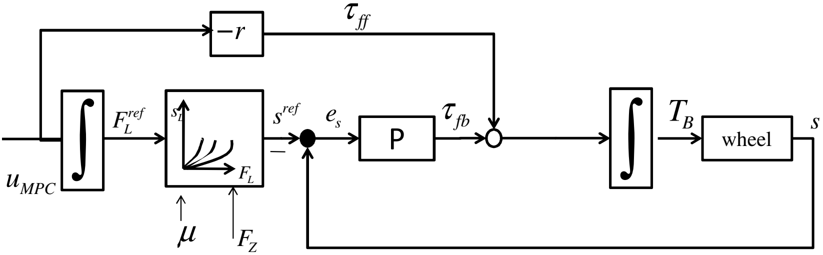
Hence, in order to design the longitudinal slip controller, we consider the model

$$\dot{s} = \left(\frac{r}{vJ_w}\right)\left(-rf_L\left(s\right) + T_{\text{eng}} - T_B\right). \tag{21}$$

Finally, since in this work we only deal with the braking phase with accelerator pedal released and a high gear, we neglect T_{eng} and use the model

$$\dot{s} = \left(\frac{r}{vJ_w}\right)\left(-rf_L\left(s\right) - T_B\right). \tag{22}$$

A reduced model for slip control



A slip control strategy

Goal of the slip controller is to obtain the desired longitudinal force

$$F_L^{\mathrm{ref}}(t) := F_L^{\mathrm{ref}}(0) + \int_0^t u_{\mathrm{MPC}}(\tau)\, d\tau \tag{23}$$

by applying a braking torque T_B. The desired slip s^{ref} is obtained through inversion of Pacejka's formula, computed for values of slip inside the stable interval:

$$s^{\mathrm{ref}} = f_L^{-1}(F_L^{\mathrm{ref}}). \tag{24}$$

The control action on the wheel is given by

$$\dot{T}_B = \tau_{ff} + \tau_{fb} = \dot{T}_{B_{ff}} + \dot{T}_{B_{fb}}. \tag{25}$$

The slip control action, then, is the sum of a feedforward action τ_{ff} and a feedback action τ_{fb}.

A slip control strategy

The feedforward control law is given by

$$\begin{aligned} \dot{T}_{B_{ff}} &= \tau_{ff} = -r u_{\mathrm{MPC}}, \\ T_{B_{ff}} = -r F_L^{\mathrm{ref}} + \mathrm{const} &= -r F_L^{\mathrm{ref}}, \end{aligned} \tag{26}$$

where u_{MPC} is the control input obtained from (8). In the Equation 26 we ignore the $const$ value because the wheel slip and the longitudinal forces are continuously estimated so that, using (24), we can write

$$r f_L(s^{\mathrm{ref}}) + T_{B_{ff}} = 0. \tag{27}$$

Feedback action

Let us define the slip error as $e_s = s^{\text{ref}} - s$. Now, taking into account (23), one has

$$\begin{aligned}
\dot{s}^{\text{ref}} &= \frac{d}{dt} f_L^{-1}\left[F_L^{\text{ref}}(t)\right] \\
&= \frac{\partial}{\partial F_L} f_L^{-1}\left[F_L^{\text{ref}}(t)\right] \dot{F}_L^{\text{ref}} \\
&= \left(1/\frac{\partial}{\partial s} f_L\left(s^{\text{ref}}\right)\right) u_{\text{MPC}},
\end{aligned}$$

so that the dynamic equation of error can be written as

$$\dot{e}_s = \dot{s}^{\text{ref}} - \dot{s} = \left(1/\frac{\partial}{\partial s} f_L\left(s^{\text{ref}}\right)\right) u_{\text{MPC}} - \left(\frac{r}{vJ_w}\right)\left(-r f_L(s) - T_B\right). \tag{28}$$

The first term of RHS of equation (28) (of order of 10^{-5} in our simulations), can be neglected w.r.t. to the second one (of order 10^2).

Feedback action

Thus, Equation 28 becomes

$$\begin{aligned}
\dot{e}_s &= \left(\frac{r}{vJ_w}\right)\left(rf_L\left(s\right)+T_B\right)\\
&\cong \left(\frac{r}{vJ_w}\right)\left[rf_L\left(s^{\text{ref}}\right)-r\frac{\partial}{\partial s}f_L\left(s^{\text{ref}}\right)e_s+T_{B_{ff}}+T_{B_{fb}}\right]\\
&= \left(\frac{r}{vJ_w}\right)\left(-r\frac{\partial}{\partial s}f_L\left(s^{\text{ref}}\right)e_s+T_{B_{fb}}\right)\\
&= -(a_1a_2)e_s+a_1T_{B_{fb}},
\end{aligned} \tag{29}$$

where the term $rf_L\left(s^{\text{ref}}\right)+T_{B_{ff}}$ is zero in view of (27) and

$$a_1=\frac{r}{vJ_w},\quad a_2=r\frac{\partial}{\partial s}f_L(s^{\text{ref}}),\quad \dot{T}_{B_{fb}}=\tau_{fb}, \tag{30}$$

with $a_2>0$ if s^{ref} is in the stable interval.

Feedback action

The transfer function $V(p; s^{\text{ref}})$, between feedback control input τ_{fb} and slip ratio error e_s, is given by

$$V(p; s^{\text{ref}}) = \frac{a_1}{p\,(p + a_1 a_2)}. \tag{31}$$

We use a proportional regulator whose gain depends on a_1 and a_2

$$\tau_{fb} = -k_p e_s. \tag{32}$$

The transfer function of the closed loop system $V_o(s)$ is given by

$$V_o(p) = \frac{a_1 k_p}{p^2 + a_1 a_2 p + a_1 k_p}. \tag{33}$$

Feedback action

The bandwidth of (33) has to be kept at least one order of magnitude less than the sampling frequency of the regulator f_s (here $f_s = 1000[Hz]$). So we choose the parameters of regulator by imposing

$$a_1 k_p = \left(\frac{2\pi}{10} f_s\right)^2 \tag{34}$$

so that

$$k_p = \frac{1}{a_1}\left(\frac{2\pi}{10} f_s\right)^2. \tag{35}$$

Equation 33 becomes

$$V_o(p) = \frac{\left(\frac{2\pi}{10} f_s\right)^2}{p^2 + a_1 a_2 p + \left(\frac{2\pi}{10} f_s\right)^2}. \tag{36}$$

Simulations show that the above closed loop transfer function depends mildly on the operating conditions: vehicle speed v, vertical force F_z and friction coefficient μ.

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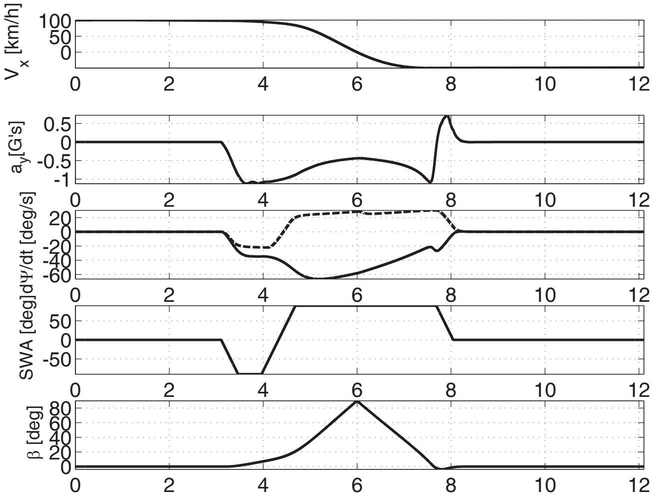
Test manoeuvre

In this manoeuver the driver:

1. turns the steering wheel from $-90\mathrm{deg}$ to $+90\mathrm{deg}$,
2. decides the initial speed (in our simulations, 120 km/h for the oversteering car and 90 km/h for the understeering car),
3. releases the accelerator pedal when the manoeuvre starts.

Simulation results



Simulation results

We tested our strategy on an oversteering sport car simulated through an ELASIS-CRF (Fiat group) proprietary simulator and an understeering light car simulated through Carsim®. The tuning parameters are:

1. sampling time $T_s = 20$ [ms];
2. the prediction horizon $H_p = 5$;
3. the control horizon $H_c = 3$;
4. the control weight for the side slip angle β: q_1=10 for the oversteering car and q_1=5 for the light car;

Simulation results

1. the control weight matrix R for the variation of longitudinal forces: $\mathrm{diag}\left[\frac{10^{-11}|\mathrm{maxF_{z_{ij}}}|}{|\mathrm{F_{z_{FL}}}|}, \frac{10^{-11}|\mathrm{maxF_{z_{ij}}}|}{|\mathrm{F_{z_{FR}}}|}, \frac{10^{-11}|\mathrm{maxF_{z_{ij}}}|}{|\mathrm{F_{z_{RL}}}|}, \frac{10^{-11}|\mathrm{maxF_{z_{ij}}}|}{|\mathrm{F_{z_{RR}}}|}\right]$ for the oversteering car and $\mathrm{diag}\left[\frac{10^{-8}|\mathrm{maxF_{z_{ij}}}|}{|\mathrm{F_{z_{FL}}}|}, \frac{10^{-8}|\mathrm{maxF_{z_{ij}}}|}{|\mathrm{F_{z_{FR}}}|}, \frac{10^{-8}|\mathrm{maxF_{z_{ij}}}|}{|\mathrm{F_{z_{RL}}}|}, \frac{10^{-8}|\mathrm{maxF_{z_{ij}}}|}{|\mathrm{F_{z_{RR}}}|}\right]$ for the light car, where the $|F_{z_{\max}}|$ is the maximum normal forces of four wheels.
2. the deactivation time $T_{\mathrm{del}} = 0.12$ [s], the side slip error activation threshold $e_{\tilde{\beta}}^{\mathrm{on}} = 0.5$ [deg] and deactivation threshold is $e_{\tilde{\beta}}^{\mathrm{off}} = 0.75 e_{\tilde{\beta}}^{\mathrm{on}}$.

The choice of the horizons length is a compromise between computational load and necessary information for the prediction model.

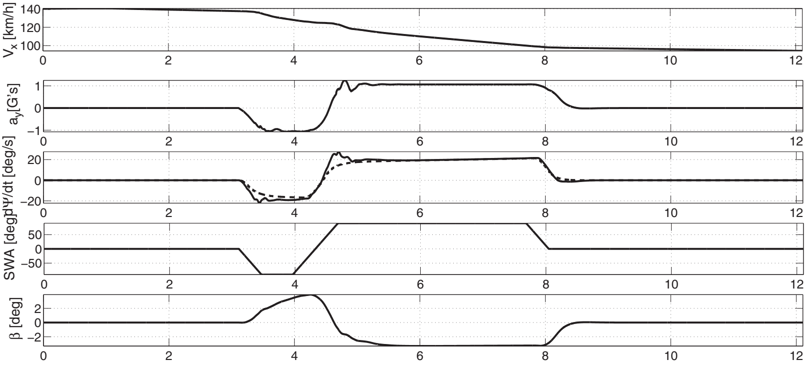
Simulation results

We present results and performance at 140 [km/h] for the oversteering sport car and at 90 [km/h] for the understeering light car; both cars, when performing this manoeuvre at 100 [km/h] (oversteering sport car) and 90 [km/h] (understeering light car), lose stability. This is illustrated in next figure for the oversteering sport car, where one can notice that β exceeds 80; this behavior is due to the oversteering characteristics, the rear traction, and the excitation of nonlinear dynamics.

In the next figure we show the value of

1. longitudinal speed v_x in $\mathrm{km/h}$;
2. lateral acceleration a_y in $\mathrm{g's}$;
3. car yaw rate (solid line) and reference yaw rate (dashed line) in $\mathrm{deg/s}$;
4. steering wheel angle SWA in deg;
5. side slip angle β in deg.

Simulation results



Simulation results