POLITECNICO DI MILANO

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Longitudinal dynamics: slip control

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"Single-corner" model

The following slides are adapted from those used in the course ''Automazione nei mezzi di trasporto''
(M.Sc. course at the Politecnico di Milano, Prof. Sergio M. Savaresi)

$$\begin{cases} J\dot{\omega} = rF_x - T_b \\ m\dot{v} = -F_x \end{cases}$$

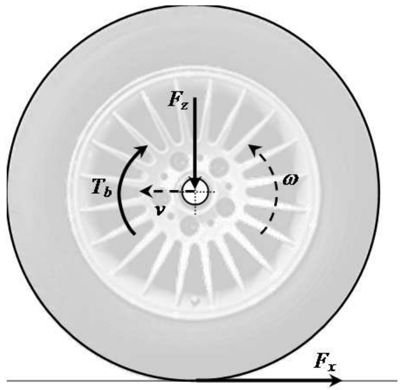
$$F_x = F_z \mu(\lambda, \cancel{\alpha; \theta_r})$$

$$\lambda = (v - \omega r)/v = 1 - r\omega / v$$

$$\begin{cases} J\dot{\omega} = rF_z \mu\left(\dfrac{v-\omega r}{v}\right) - T_b \\ m\dot{v} = -F_z \mu\left(\dfrac{v-\omega r}{v}\right) \end{cases}$$

- ω : angular speed of the wheel ([rad/s]; $\omega > 0$ is assumed);
- v : longitudinal speed of the vehicle body;
- T_b : braking torque (control/input variable);
- F_x: longitudinal road-tire contact force;
- F_z : vertical road-tire contact force;
- J, m and r are the momentum of inertia of the wheel, the quarter-car mass, and the wheel radius

(e.g. $J = 1 Kg\, m^2$, $m = 225 Kg$, $r = 0.28m$).



SISO system, 2nd order (n= 2), non-linear, time-invariant

Change of state variables

$$\begin{cases} J\dot{\omega} = rF_z\mu\left(\dfrac{v-\omega r}{v}\right) - T_b \\ m\dot{v} = -F_z\mu\left(\dfrac{v-\omega r}{v}\right) \end{cases}$$

$$\dot{\lambda} = -\frac{r}{v}\dot{\omega} + \frac{r\omega}{v^2}\dot{v}$$

$$\omega = \frac{v}{r}(1-\lambda)$$

CHANGE OF STATE VARIABLES

$$\begin{cases} \dot{\lambda} = -\dfrac{1}{\boxed{v}}\left(\dfrac{(1-\lambda)}{m} + \dfrac{r^2}{J}\right)F_z\mu(\lambda) + \dfrac{r}{\boxed{v}J}T_b \\ \boxed{m\dot{v} = -F_z\mu(\lambda)} \end{cases}$$

SISO system, 2nd order (n= 2), non-linear, time-invariant, strictly proper (or: SISO, non-linear, 1st order, time-varying)

Equilibrium points (I)

$$\begin{cases} \dot{\lambda} = -\dfrac{1}{v}\left(\dfrac{(1-\lambda)}{m} + \dfrac{r^2}{J}\right) F_z \mu(\lambda) + \dfrac{r}{vJ} T_b \\ m\dot{v} = -F_z \mu(\lambda) \end{cases}$$

Linear, **normalized**, wheel **deceleration**

$$\eta = -\frac{\dot{\omega} r}{g}$$

$$\dot{\lambda} = 0, \quad \eta = \bar{\eta}$$

not $\dot{v} = 0$

$$\bar{T}_b = F_z \left(r + \frac{J}{rm}(1-\bar{\lambda}) \right) \mu(\bar{\lambda})$$

Note: DO NOT depend on v

Equilibrium points

$$\begin{cases} \dot{\lambda} = -\dfrac{1}{v}\left(\dfrac{(1-\lambda)}{m} + \dfrac{r^2}{J}\right) F_z \mu(\lambda) + \dfrac{r}{vJ} T_b \\ m\dot{v} = -F_z \mu(\lambda) \end{cases}$$

$$\dot{\lambda} = 0, \quad \eta = \bar{\eta}$$

$$\boxed{\text{not} \quad \dot{v} = 0}$$

$$\lambda = 1 - r\omega / v$$

$$(\dot{\omega}v - \dot{v}\omega)/v^2 = 0$$

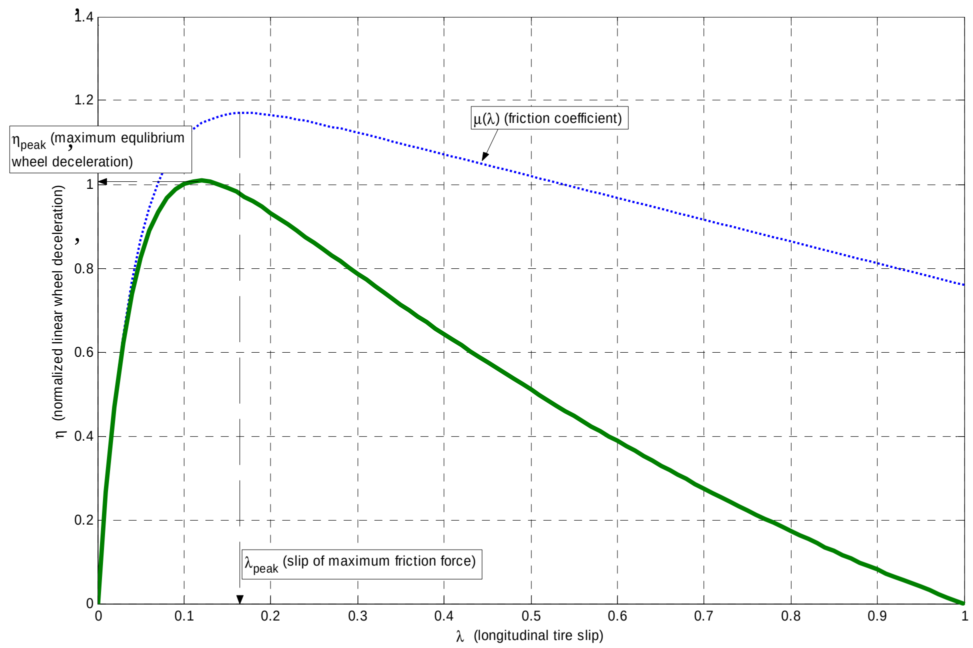
$$\boxed{\dot{\omega} = \omega\dot{v}/v}$$

$$\omega = v(1-\lambda)/r$$

$$m\dot{v} = -F_z \mu(\lambda)$$

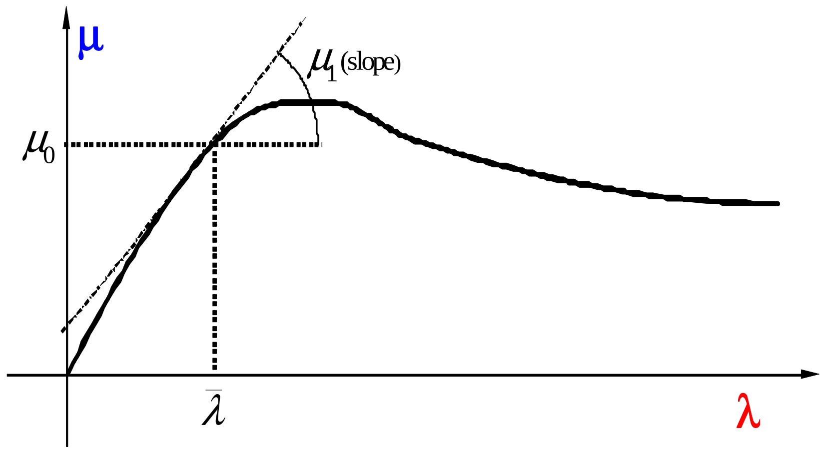
$$\eta = -\dot{\omega}r/g$$

$$\bar{\eta} = (1-\bar{\lambda})\frac{F_z}{mg}\mu(\bar{\lambda})$$



Linearization model (method I)

$$\delta T_b = T_b - \bar{T}_b \qquad \delta\lambda = \lambda - \bar{\lambda}$$



$$\mu_1 = \mu_1(\bar{\lambda}) = \partial\mu(\lambda)/\partial\lambda\big|_{\lambda=\bar{\lambda}}$$
$$\mu_0 = \mu(\bar{\lambda})$$

$$\begin{cases} \boxed{\dot{\lambda} = -\dfrac{1}{v}\left(\dfrac{(1-\lambda)}{m} + \dfrac{r^2}{J}\right)F_z\mu(\lambda) + \dfrac{r}{vJ}T_b} \\ m\dot{v} = -F_z\mu(\lambda) \end{cases}$$

Assumption: v dynamics (Body Dynamics) are much slower than λ or ω dynamics (Wheel Dynamics).
Consider *v* as slowly varying.

Linearization model (method I)

Equation of λ:

$$\dot{\lambda}=-\frac{1}{v}\left(\frac{(1-\lambda)}{m}+\frac{r^2}{J}\right)F_z\mu(\lambda)+\frac{r}{vJ}T_b$$

$$\dot{\lambda}=-\frac{F_z}{v}\left(\frac{(1-\lambda)}{m}\mu(\lambda)+\frac{r^2}{J}\mu(\lambda)\right)+\frac{r}{vJ}T_b$$

:

Linearizing around $\bar{\lambda},\bar{T}_b$

$$\delta\dot{\lambda}=-\frac{F_z}{v}\left(\frac{-1}{m}\mu_0+\frac{(1-\bar{\lambda})}{m}\mu_1+\frac{r^2}{J}\mu_1\right)\delta\lambda+\frac{r}{vJ}\delta T_b$$

$$\delta\dot{\lambda}=-\frac{F_z}{v}\left(\left(\frac{(1-\bar{\lambda})}{m}+\frac{r^2}{J}\right)\mu_1-\frac{\mu_0}{m}\right)\delta\lambda+\frac{r}{vJ}\delta T_b$$

$$\boxed{G_\lambda(s)=\frac{\left[\frac{r}{vJ}\right]}{s+\left[\frac{F_z}{v}\left(\left(\frac{(1-\bar{\lambda})}{m}+\frac{r^2}{J}\right)\mu_1-\frac{\mu_0}{m}\right)\right]}}$$

Linearization model (method I)

$$J\dot{\omega} = rF_z\mu(\lambda) - T_b \qquad -\frac{r}{g}J\dot{\omega} = -\frac{r}{g}\left(rF_z\mu(\lambda) - T_b\right) \qquad J\eta = -\frac{r^2}{g}F_z\mu(\lambda) + \frac{r}{g}T_b$$

$$\boxed{\eta = -\frac{r^2}{Jg}F_z\mu(\lambda) + \frac{r}{Jg}T_b}$$

$$\delta\eta = -\frac{r^2F_z}{Jg}\mu_1\delta\lambda + \frac{r}{Jg}\delta T_b \qquad G_\eta(s) = -\frac{r^2F_z}{Jg}\mu_1 G_\lambda(s) + \frac{r}{Jg} = \frac{r}{Jg}\left\{1 - \frac{rF_z\mu_1\left[\frac{r}{vJ}\right]}{s + \left[\frac{F_z}{v}\left(\left(\frac{(1-\bar{\lambda})}{m} + \frac{r^2}{J}\right)\mu_1 - \frac{\mu_0}{m}\right)\right]}\right\}$$

$$G_\eta(s) = \frac{r}{Jg}\frac{s + \left[\frac{F_z}{v}\left(\left(\frac{(1-\bar{\lambda})}{m} + \frac{r^2}{J}\right)\mu_1 - \frac{\mu_0}{m}\right)\right] - rF_z\mu_1\left[\frac{r}{vJ}\right]}{s + \left[\frac{F_z}{v}\left(\left(\frac{(1-\bar{\lambda})}{m} + \frac{r^2}{J}\right)\mu_1 - \frac{\mu_0}{m}\right)\right]}$$

$$G_\eta(s) = \frac{r}{Jg}\frac{s + \frac{F_z}{v}\left[\left(\frac{(1-\bar{\lambda})}{m} + \frac{r^2}{J}\right)\mu_1 - \frac{\mu_0}{m} - \mu_1\frac{r^2}{J}\right]}{s + \left[\frac{F_z}{v}\left(\left(\frac{(1-\bar{\lambda})}{m} + \frac{r^2}{J}\right)\mu_1 - \frac{\mu_0}{m}\right)\right]}$$

$$\boxed{G_\eta(s) = \frac{r}{Jg}\frac{s + \frac{F_z}{mv}\left[(1-\bar{\lambda})\mu_1 - \mu_0\right]}{s + \left[\frac{F_z}{v}\left(\left(\frac{(1-\bar{\lambda})}{m} + \frac{r^2}{J}\right)\mu_1 - \frac{\mu_0}{m}\right)\right]}}$$

Linearization model (method II)

Alternative method of linearization

Do not make assumptions on the dynamic decoupling between the wheel and body.
Linearize around a **point (non-equilibrium)** defined by

$$\bar{v} \quad \bar{\omega} \quad \bar{\lambda} = (\bar{v} - \bar{\omega} r)/\bar{v}$$

$$\mu\left(\lambda(v,\omega)\right) = \mu\left(\bar{\lambda}\right) + \left.\frac{\partial \mu}{\partial v}\right|_{\lambda=\bar{\lambda}} (v - \bar{v}) + \left.\frac{\partial \mu}{\partial \omega}\right|_{\lambda=\bar{\lambda}} (\omega - \bar{\omega}) + O((v-\bar{v})^2, (\omega-\bar{\omega})^2)$$

,

$$\mu\left(\lambda(v,\omega)\right) \approx \mu\left(\bar{\lambda}\right) + \left.\left[\frac{\partial \mu}{\partial \lambda}\frac{\partial \lambda}{\partial v}\right]\right|_{\lambda=\bar{\lambda}} (v - \bar{v}) + \left.\left[\frac{\partial \mu}{\partial \lambda}\frac{\partial \lambda}{\partial \omega}\right]\right|_{\lambda=\bar{\lambda}} (\omega - \bar{\omega})$$

$$\mu(\lambda) \approx \mu_0 + \mu_1 \frac{\bar{\omega} r}{\bar{v}^2} \delta v - \mu_1 \frac{r}{\bar{v}} \delta\omega$$

Linearization model (method II)

$$\begin{cases} J\dot{\omega} = rF_z\left[\mu_0 + \mu_1 \dfrac{\overline{\omega} r}{\overline{v}^2}\delta v - \mu_1 \dfrac{r}{\overline{v}}\delta\omega\right] - T_b \\ m\dot{v} = -F_z\left[\mu_0 + \mu_1 \dfrac{\overline{\omega} r}{\overline{v}^2}\delta v - \mu_1 \dfrac{r}{\overline{v}}\delta\omega\right] \end{cases}$$

, $$\begin{cases} J\dot{\overline{\omega}} = rF_z[\mu_0] - \overline{T}_b \\ m\dot{\overline{v}} = -F_z[\mu_0] \end{cases}$$

equilibrium

$$\begin{cases} \delta\dot{\omega} = \left[-\mu_1 \dfrac{r^2 F_z}{\overline{v}J}\right]\delta\omega + \left[\mu_1 \dfrac{\overline{\omega} r^2}{J\overline{v}^2}F_z\right]\delta v - \dfrac{1}{J}\delta T_b \\ \delta\dot{v} = \left[\mu_1 \dfrac{r}{m\overline{v}}F_z\right]\delta\omega + \left[-\mu_1 \dfrac{\overline{\omega} r}{m\overline{v}^2}F_z\right]\delta v \end{cases}$$

Is the final relation identical to the previous one ...?

Linearization model (method II)

$$
\begin{cases}
\delta\dot{\omega} = \left[-\mu_1 \dfrac{r^2 F_z}{\bar{v} J}\right]\delta\omega + \left[\mu_1 \dfrac{\bar{\omega} r^2}{J \bar{v}^2} F_z\right]\delta v - \dfrac{1}{J}\delta T_b \\[2ex]
\delta\dot{v} = \left[\mu_1 \dfrac{r}{m\bar{v}} F_z\right]\delta\omega + \left[-\mu_1 \dfrac{\bar{\omega} r}{m\bar{v}^2} F_z\right]\delta v
\end{cases}
$$

```
>> A=sym('[-m1*r^2*Fz/(v*J),m1*w*r^2*Fz/(J*v^2);m1*r*Fz/(v*m),-m1*w*r*Fz/(m*v^2)]');
>> B=sym('[-1/J;0]');
>> C=sym('[1,0;0,1]');
>> I=sym(eye(2));
>> F=C*inv(sym('s')*I-A)*B;
>> pretty(F)

                  [                   2                       ]
                  [             s m v   + m1 w r Fz           ]
                  [- -----------------------------------------]
                  [          2                      2         ]
                  [  s (s v  J m + J m1 w r Fz + m1 r  Fz m v)]
                  [                                           ]
                  [              m1 r Fz v                    ]
                  [- -----------------------------------------]
                  [          2                      2         ]
                  [  s (s v  J m + J m1 w r Fz + m1 r  Fz m v)]
```

$$
G_\omega(s) = -\frac{1}{Js}\,\frac{s + \dfrac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}(1-\bar{\lambda})}{s + \left[\dfrac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda}) + \dfrac{mr^2}{J}\right)\right]}
\qquad
G_v(s) = -\frac{1}{Js}\,\frac{\dfrac{\mu_1(\bar{\lambda})rF_z}{m\bar{v}}}{s + \left[\dfrac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda}) + \dfrac{mr^2}{J}\right)\right]}
$$

Linearization model (method II)

$$\delta\lambda = -\frac{r}{\bar{v}}\delta\omega + \frac{\bar{\omega}r}{\bar{v}^2}\delta v$$

$$\boxed{G_\lambda(s) = \frac{\left[\dfrac{r}{vJ}\right]}{s + \left[\dfrac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda}) + \dfrac{mr^2}{J}\right)\right]}}$$

$$\Updownarrow$$

$$G_\lambda(s) = \frac{\left[\dfrac{r}{vJ}\right]}{s + \left[\dfrac{F_z}{v}\left(\left(\dfrac{(1-\bar{\lambda})}{m} + \dfrac{r^2}{J}\right)\mu_1 - \dfrac{\mu_0}{m}\right)\right]}$$

Linearization model

Method I	Method II
$G_\lambda(s)=\dfrac{\left[\dfrac{r}{vJ}\right]}{s+\left[\dfrac{F_z}{v}\left(\left(\dfrac{(1-\bar{\lambda})}{m}+\dfrac{r^2}{J}\right)\mu_1-\dfrac{\mu_0}{m}\right)\right]}$	$G_\lambda(s)=\dfrac{\left[\dfrac{r}{vJ}\right]}{s+\left[\dfrac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda})+\dfrac{mr^2}{J}\right)\right]}$
$G_\eta(s)=\dfrac{r}{Jg}\dfrac{s+\dfrac{F_z}{mv}\left[(1-\bar{\lambda})\mu_1-\mu_0\right]}{s+\left[\dfrac{F_z}{v}\left(\left(\dfrac{(1-\bar{\lambda})}{m}+\dfrac{r^2}{J}\right)\mu_1-\dfrac{\mu_0}{m}\right)\right]}$	$G_\eta(s)=\dfrac{r}{Jg}\dfrac{s+\dfrac{F_z}{m\bar{v}}(1-\bar{\lambda})\mu_1(\bar{\lambda})}{s+\left[\dfrac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda})+\dfrac{mr^2}{J}\right)\right]}$
Linearization around an equilibrium point	Linearization around a non-equilibrium point
Assumption of v-slowly-varying	No v-slowly-varying assumption

Analysis: open-loop

$$G_\lambda(s) = \frac{\left[\dfrac{r}{vJ}\right]}{s + \left[\dfrac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda}) + \dfrac{mr^2}{J}\right)\right]}$$

$$G_\eta(s) = \frac{r}{Jg}\frac{s + \dfrac{F_z}{m\bar{v}}(1-\bar{\lambda})\mu_1(\bar{\lambda})}{s + \left[\dfrac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda}) + \dfrac{mr^2}{J}\right)\right]}$$

Pole (stability):

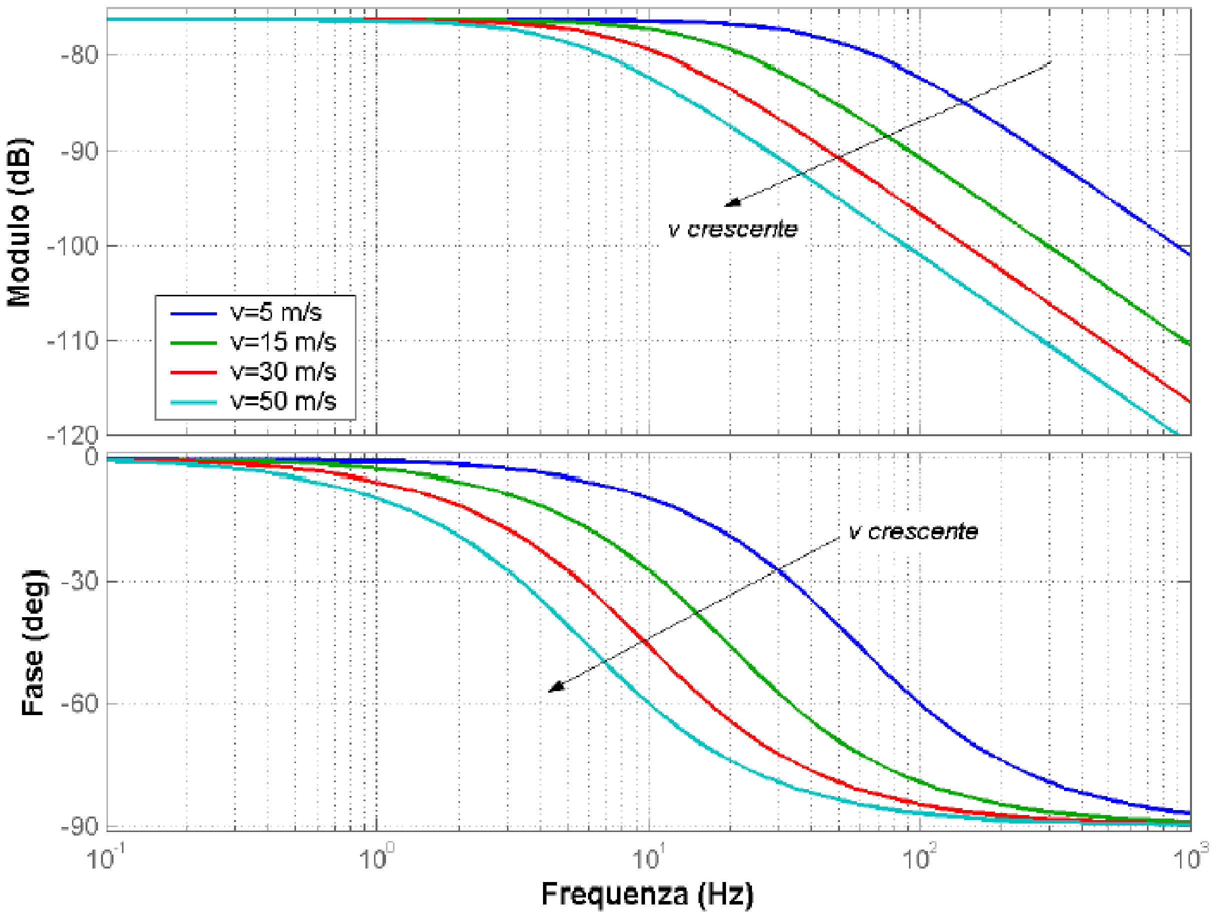
$$\frac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda}) + \frac{mr^2}{J}\right) > 0 \qquad \mu_1(\bar{\lambda}) > 0$$

Zero (minimum phase):

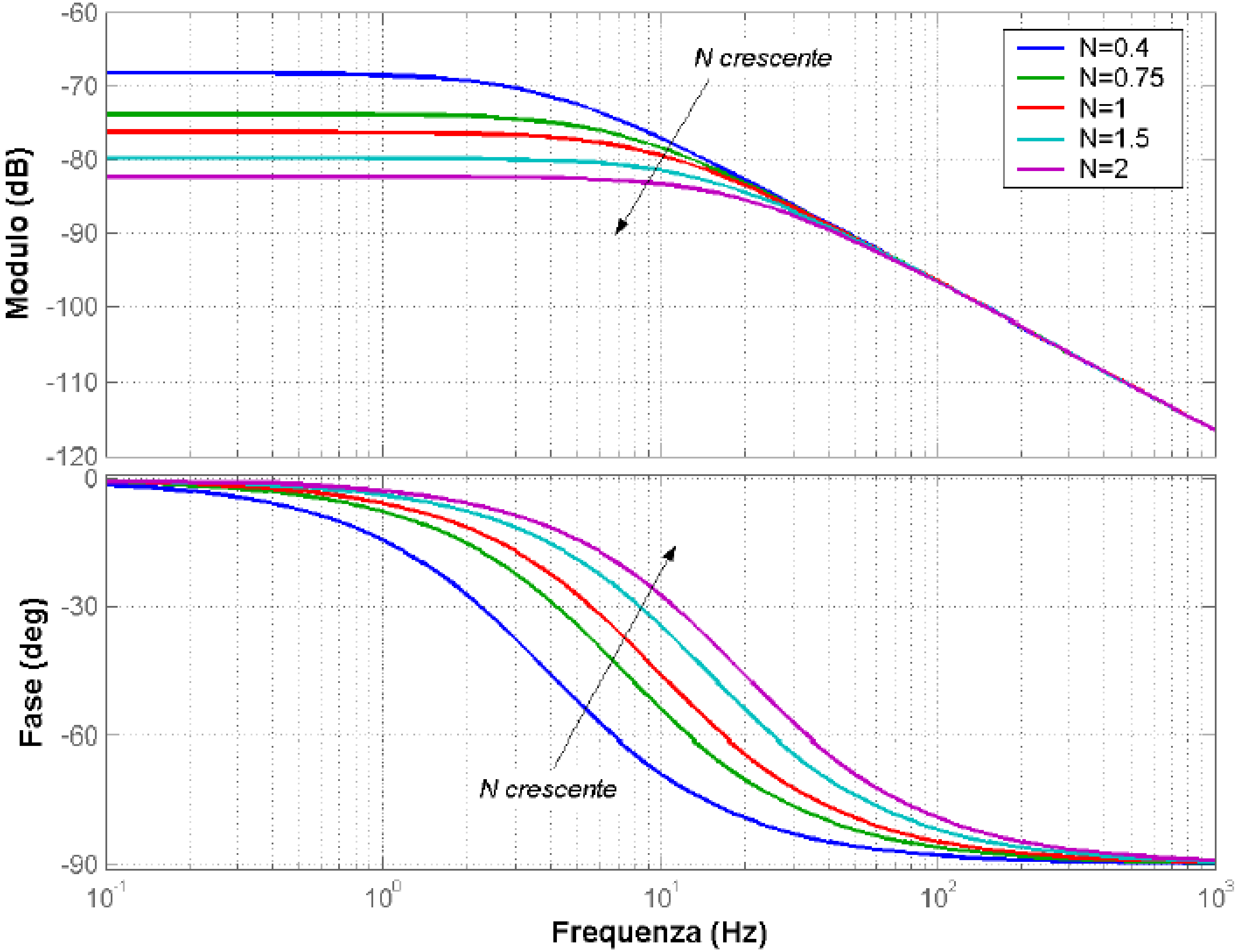
$$\frac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}(1-\bar{\lambda}) > 0 \qquad \mu_1(\bar{\lambda}) > 0$$

Sensitivity with respect to the longitudinal vehicle speed

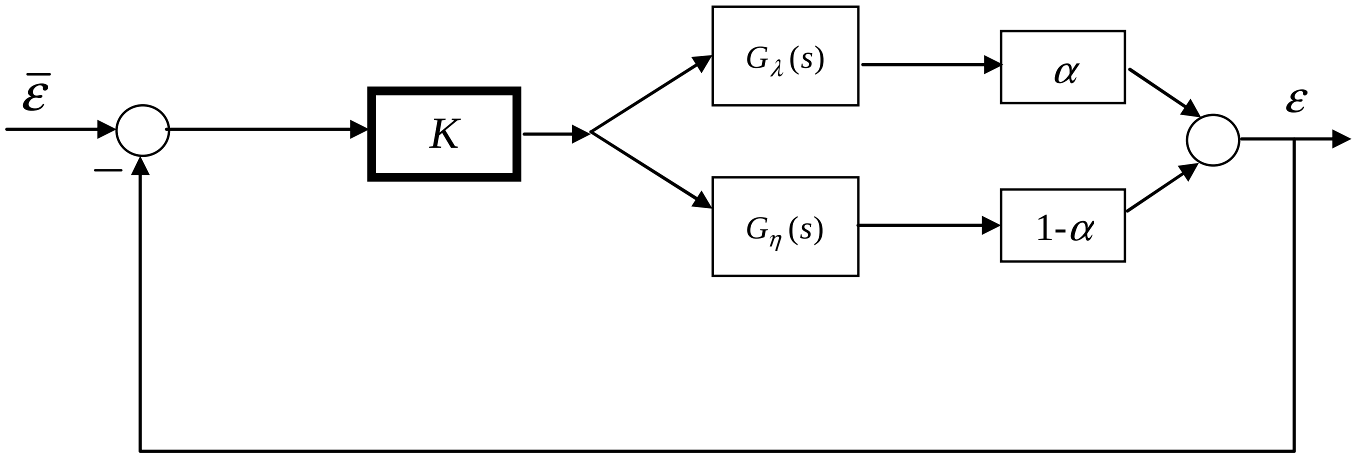
F.d.t. From T_b to λ (nominal F_z= Mg)



Sensitivity with respect to vertical load



$$N = \frac{F_z}{mg}$$

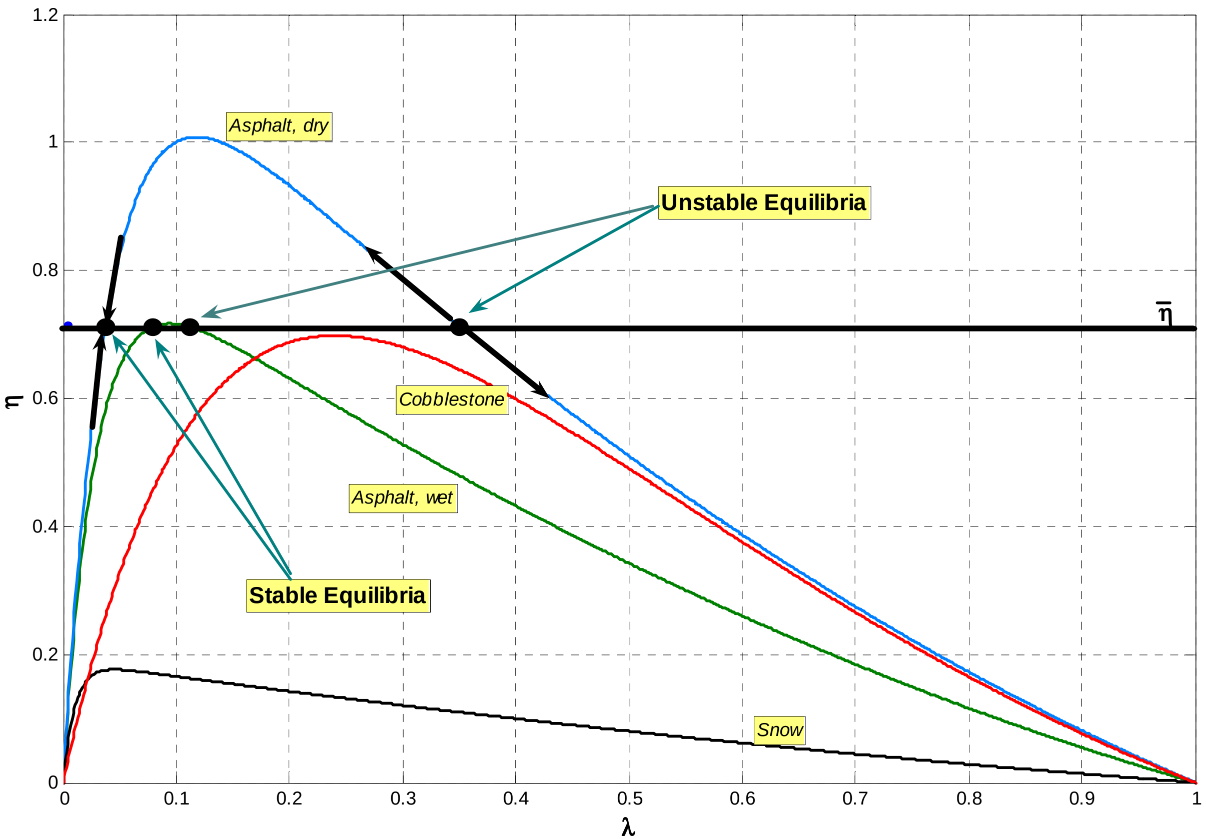


$$G_\lambda(s) = \frac{\left[\dfrac{r}{vJ}\right]}{s + \left[\dfrac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda}) + \dfrac{mr^2}{J}\right)\right]}$$

α= 1 (Slip control)
α= 0 (Deceleration control)

$$G_\eta(s) = \frac{\dfrac{r}{Jg}\left(s + \left[\dfrac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda})\right)\right]\right)}{s + \left[\dfrac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda}) + \dfrac{mr^2}{J}\right)\right]}$$

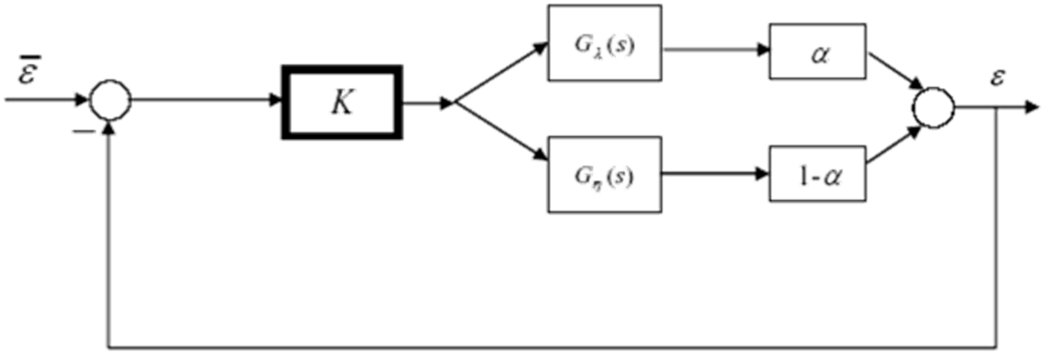
α = 0 (deceleration control)



- No fixed set-point
- No unique equilibrium

- deceleration measurement very simple and robust

$\alpha = 0$ (deceleration control)



$$G_\eta(s) = \frac{\dfrac{r}{Jg}\left(s + \left[\dfrac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda})\right)\right]\right)}{s + \left[\dfrac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda}) + \dfrac{mr^2}{J}\right)\right]}$$

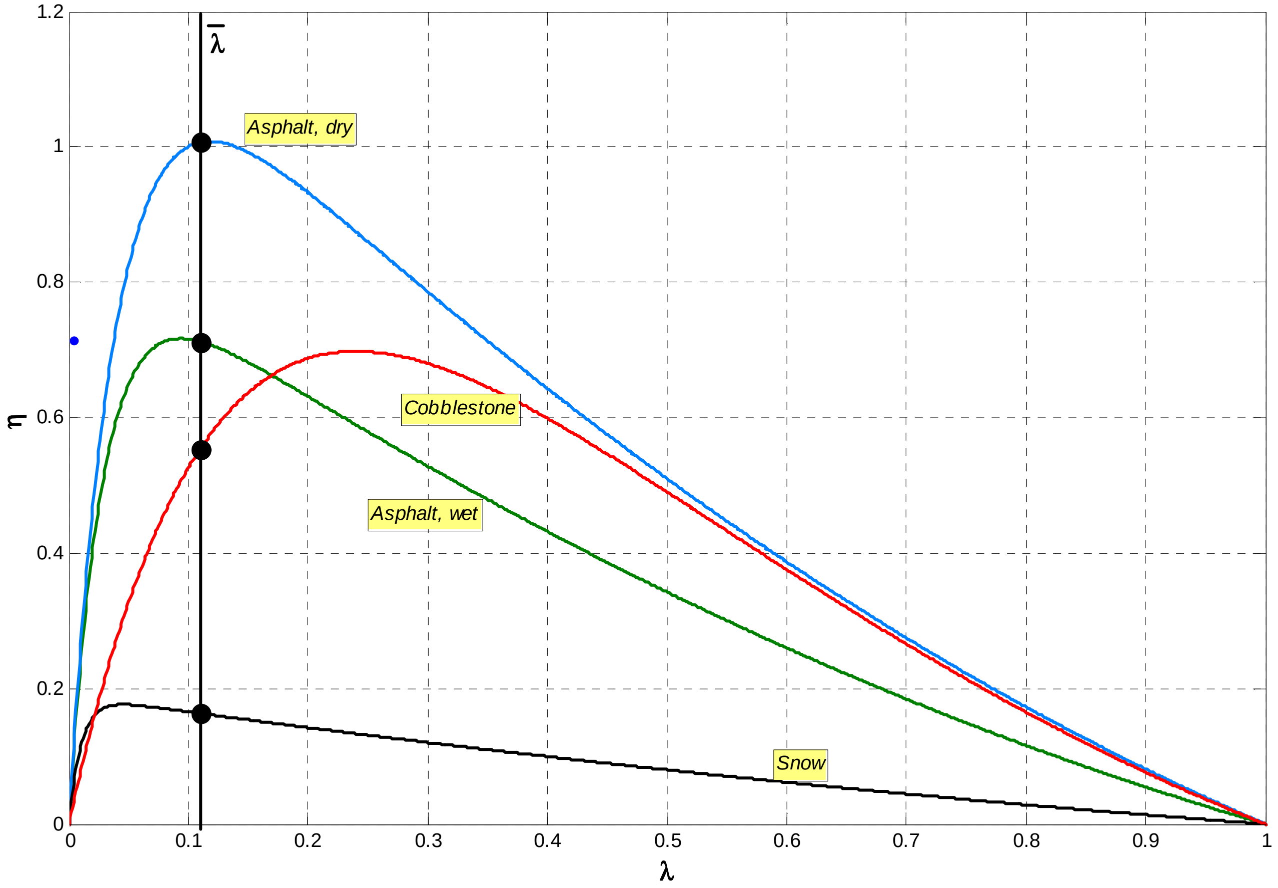
$$\chi_\eta(s) = \left(1 + K\frac{r}{Jg}\right)s + \frac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda})\left(1 + K\frac{r}{Jg}\right) + \frac{mr^2}{J}\right)$$

$$\Downarrow$$

$$\boxed{\mu_1(\bar{\lambda})\left((1-\bar{\lambda})\left(1 + K\frac{r}{Jg}\right) + \frac{mr^2}{J}\right) > 0}$$

Impossible to find a stabilizing K in all working conditions

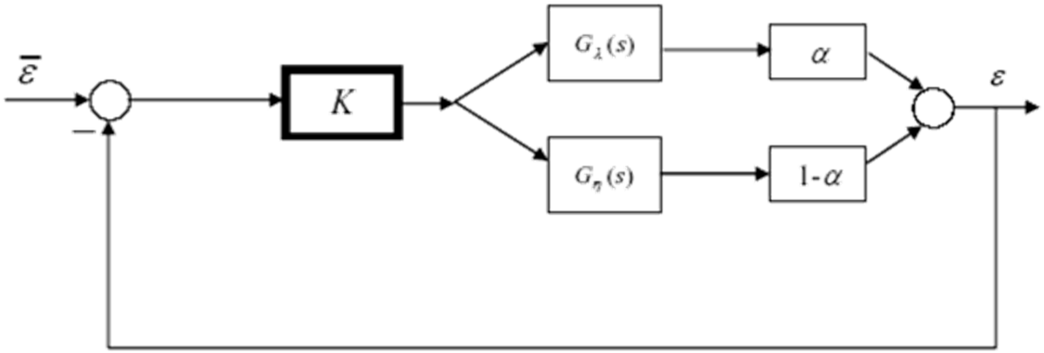
α = 1 (slip control)



- Set-point (almost) fixed
- Unique equilibrium

- Sensitivity to measurement noise (slip measurement is critical)

α = 1 (slip control)



$$G_\lambda(s) = \frac{\left[\frac{r}{vJ}\right]}{s + \left[\frac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda}) + \frac{mr^2}{J}\right)\right]}$$

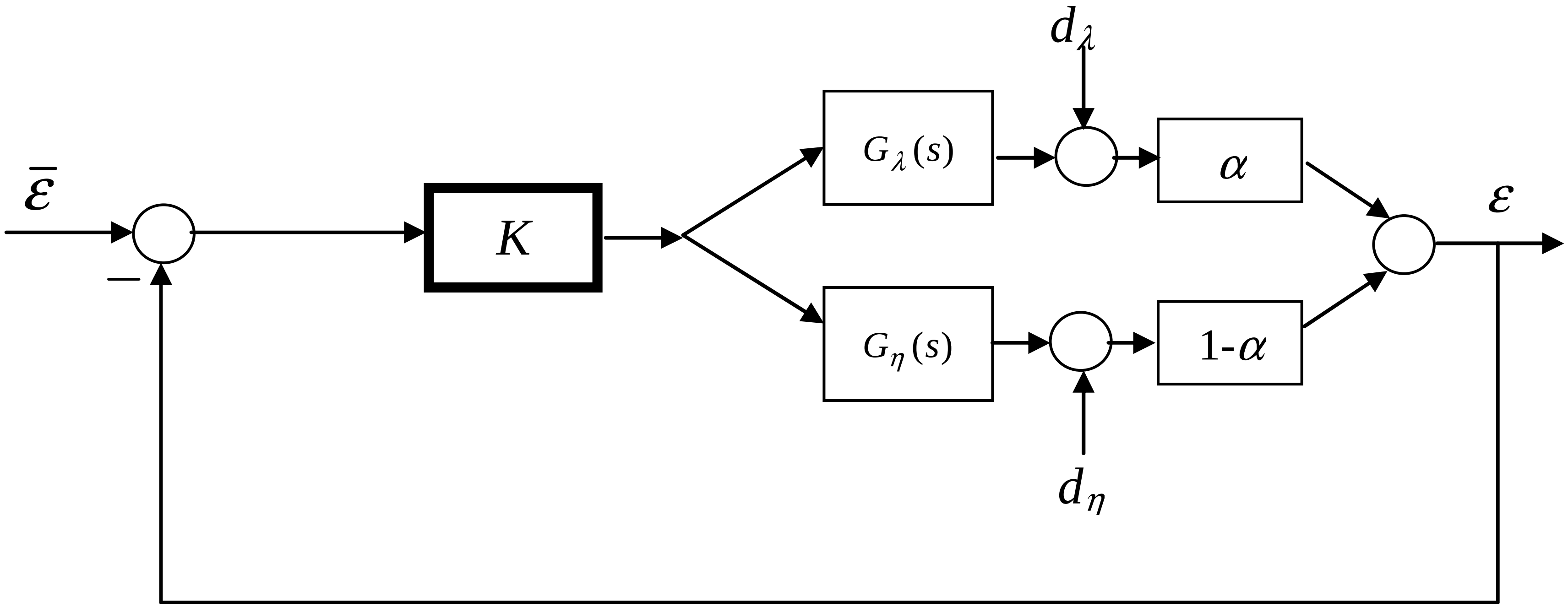
$$\chi_\lambda(s) = s + \frac{1}{\bar{v}}\left[\frac{\mu_1(\bar{\lambda})F_z}{m}\left((1-\bar{\lambda}) + \frac{mr^2}{J}\right) + K\frac{r}{J}\right]$$

$$\Downarrow$$

$$\boxed{K\frac{r}{J} > -\frac{\mu_1(\bar{\lambda})F_z}{m}\left((1-\bar{\lambda}) + \frac{mr^2}{J}\right)}$$

Possible to find a stabilizing K in all working conditions

Mixed Slip-Deceleration control (MSD-control)

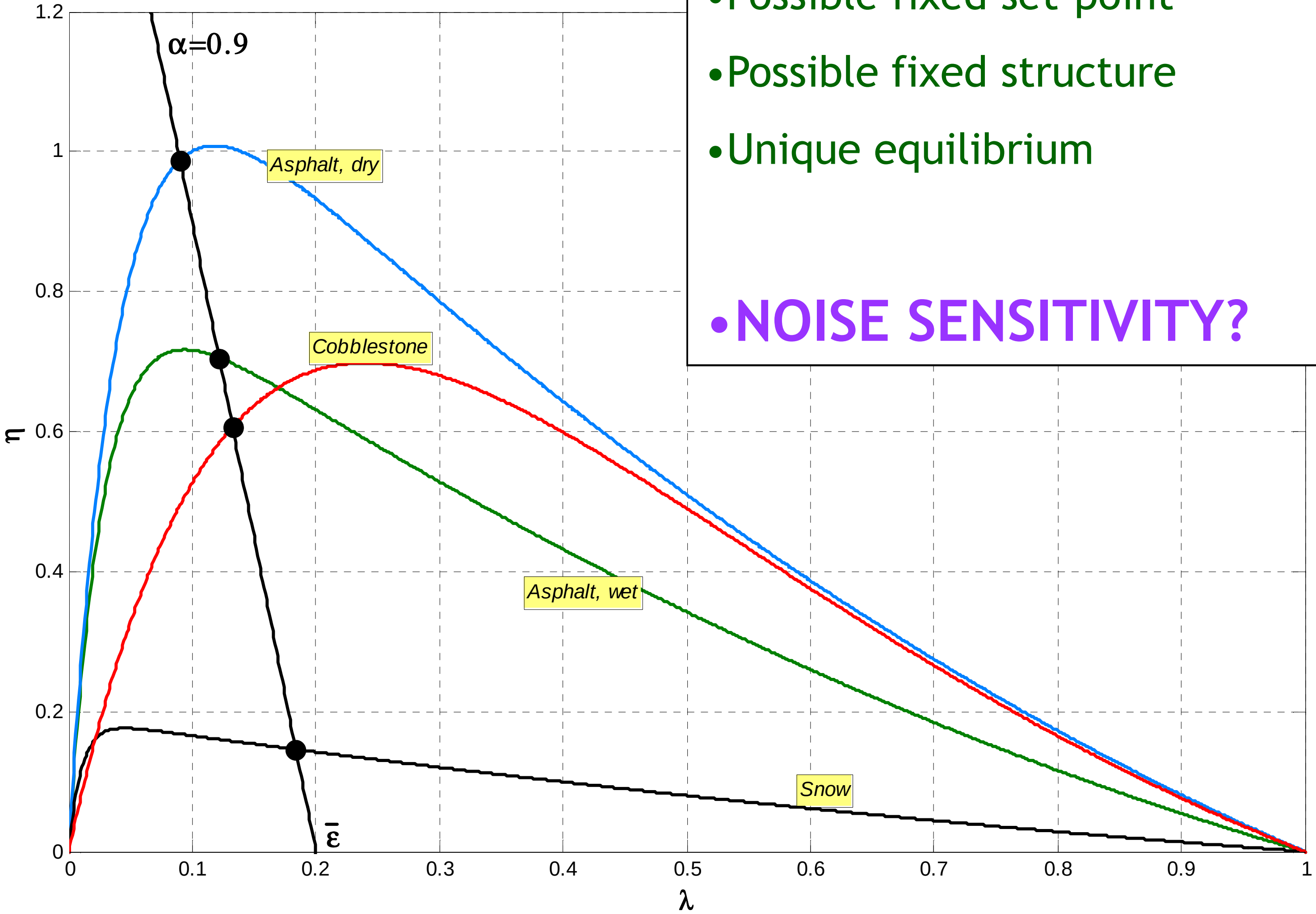


$$\boxed{\epsilon = \alpha\lambda + (1-\alpha)\eta, \qquad \alpha \in [0,1]}$$

Savaresi s.m., M. Tanelli, C. Cantoni (2007). Mixed slip-deceleration control in automotive braking systems. ASME Transactions: Journal of Dynamic Systems, Measurement and Control, Vol.129, No. 1, pp.20-31 [2008 ASME Dynamic Systems and Control Rudolf Kalman Best Paper Award]

PCT/EP2005/050820 (2005). “Sistema di controllo automatico della frenatura", Freni Brembo S.p.A. (Cantoni C., Savaresi S., Charalambakis D., Tanelli M.).

0 <α <1 (mixed slip-deceleration control - MSD-control)



- Possible fixed set-point
- Possible fixed structure
- Unique equilibrium

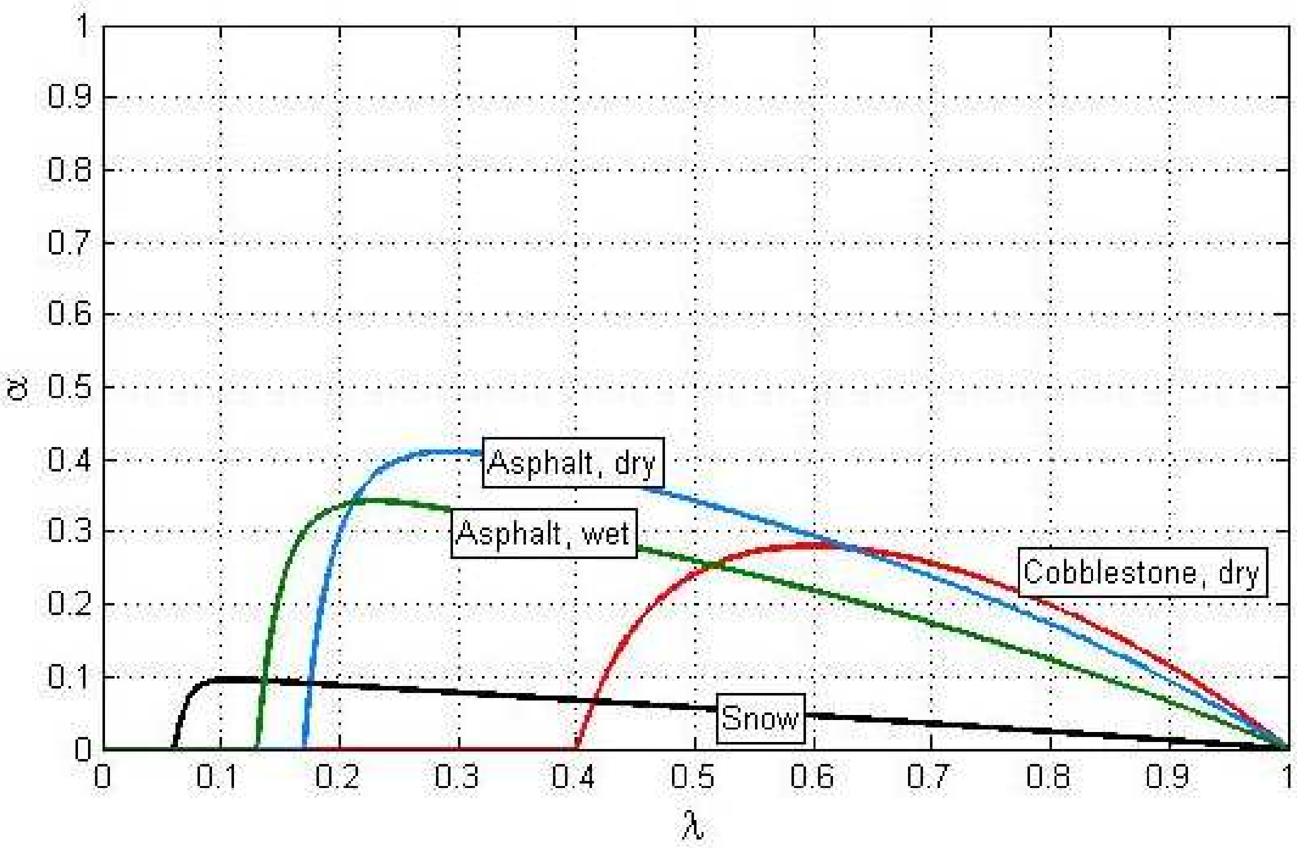
- **NOISE SENSITIVITY?**

Limits in the choice of α

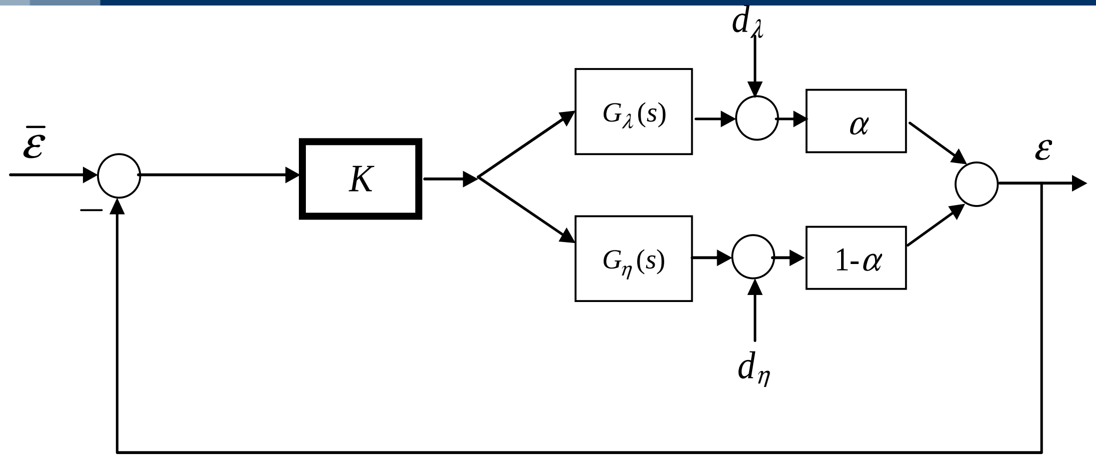
$$\alpha_{min} = \max_{\bar{\lambda},\theta_r} \frac{\mu_1(\bar{\lambda};\theta_r)(\bar{\lambda}-1)\frac{F_z}{mg}}{1+\left[\mu_1(\bar{\lambda};\theta_r)(\bar{\lambda}-1)\right]\frac{F_z}{mg}}$$

$$F_z = m\,g$$

$$\alpha_{min} \approx 0.45$$



Noise sensitivity



$$\sigma_\lambda^2 > \sigma_\eta^2$$

$$D_\varepsilon(s;\alpha) = \left(\alpha D_\lambda(s) + (1-\alpha) D_\eta(s)\right) S_\varepsilon(s;\alpha)$$

$$\gamma(\alpha) = \frac{\mathrm{var}\left[d_\varepsilon(\alpha)\right]}{\mathrm{var}\left[d_\varepsilon(1)\right]}$$

Noise sensitivity

$$\gamma(\alpha) = \frac{\mathrm{var}[d_\varepsilon(\alpha)]}{\mathrm{var}[d_\varepsilon(1)]} = \Psi(\alpha)\Phi(\alpha)$$

$$\Psi(\alpha) = \frac{\alpha^2\sigma_\lambda^2 + (1-\alpha)^2\sigma_\eta^2}{\sigma_\lambda^2}$$

$$0.80 \le \alpha \le 0.95$$

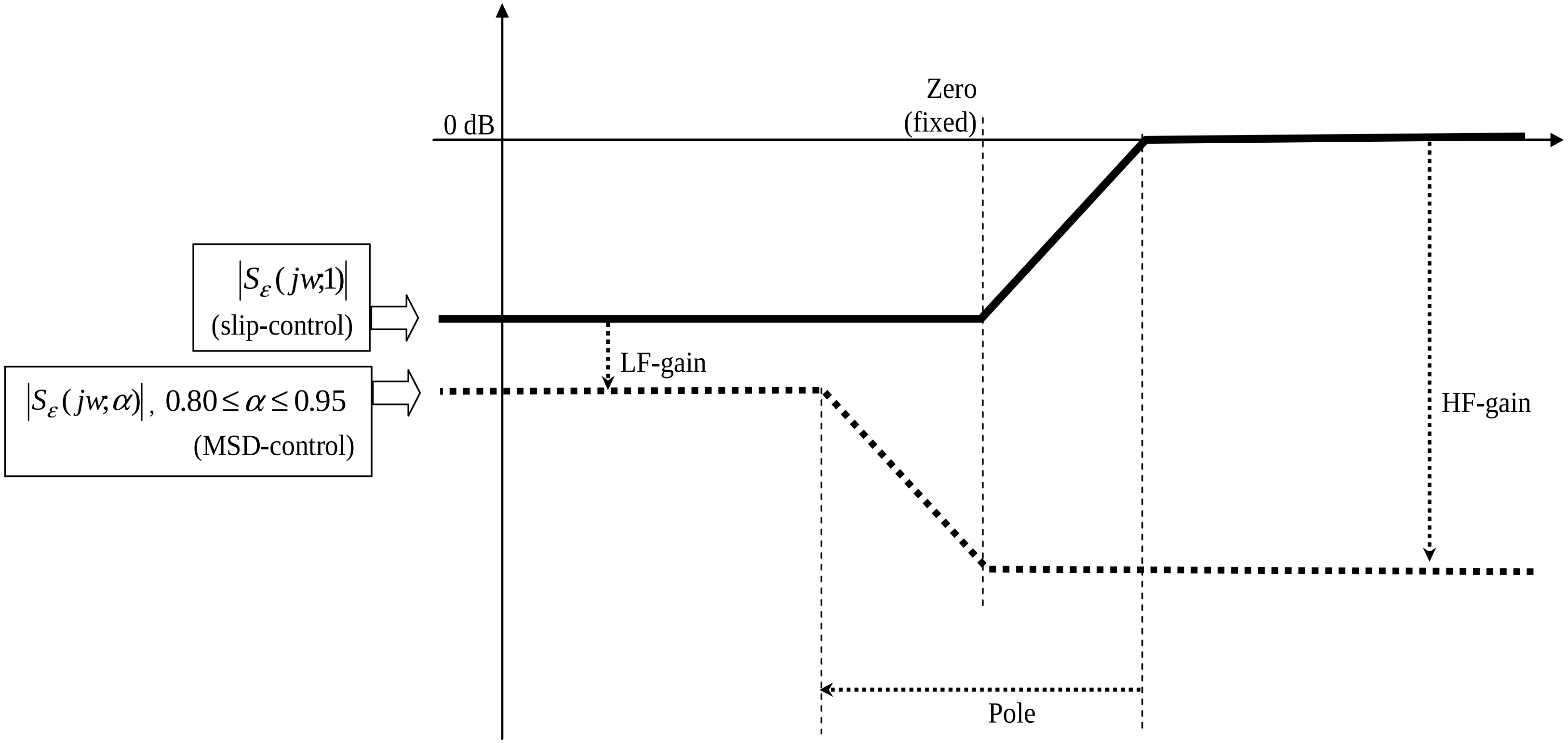
$$\sigma_\lambda^2 > \sigma_\eta^2$$

$$\Psi(\alpha) \approx \alpha^2$$

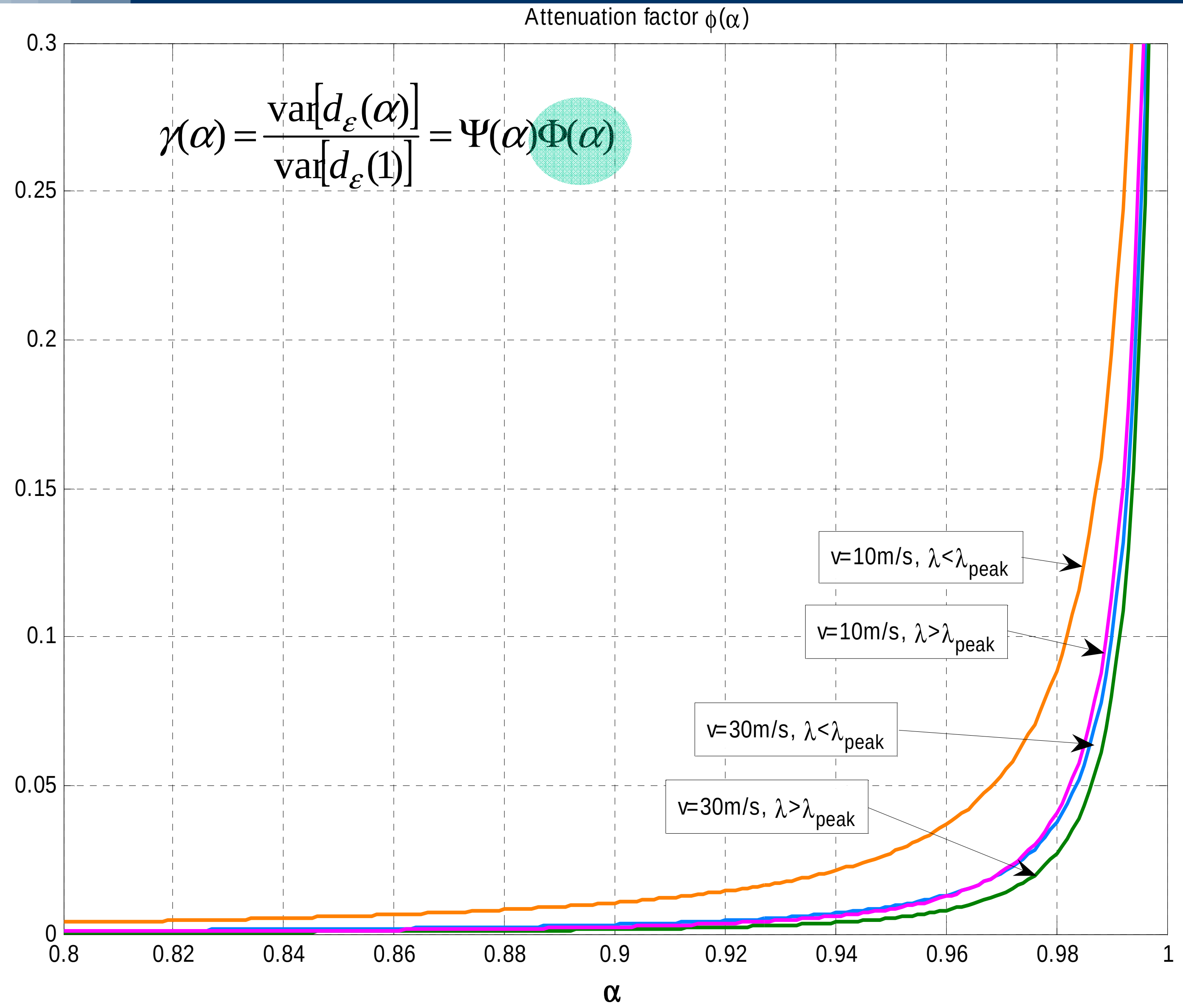
$$\Phi(\alpha) = \frac{\int_{w=0}^{\Omega_N} |S_\varepsilon(jw;\alpha)|^2 dw}{\int_{w=0}^{\Omega_N} |S_\varepsilon(jw;1)|^2 dw}$$

Noise sensitivity

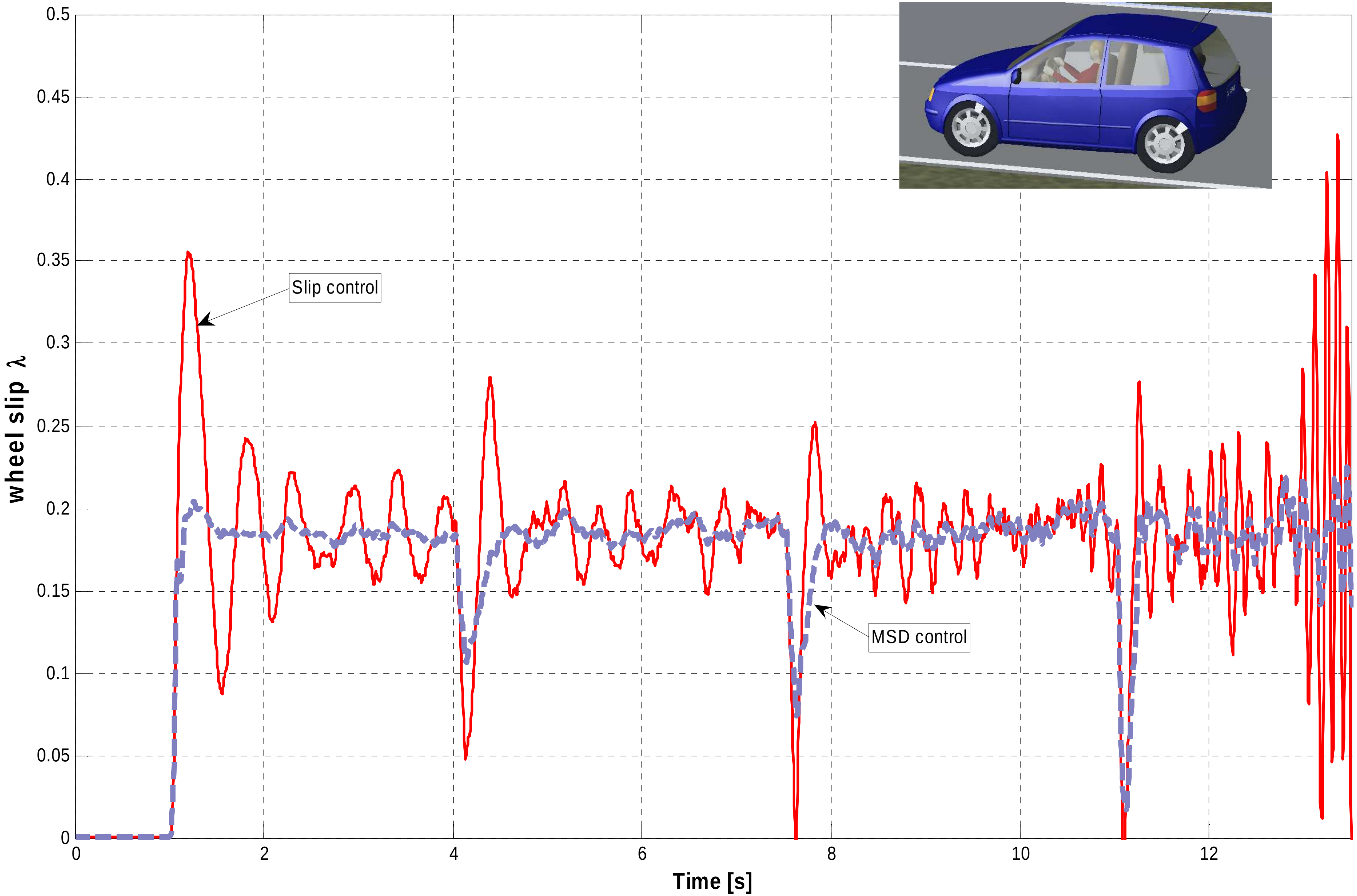
$$S_\varepsilon(s;\alpha)=\frac{s+\dfrac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda})+\dfrac{mr^2}{J}\right)}{s\left[1+K\dfrac{r}{Jg}(1-\alpha)\right]+\dfrac{1}{\bar{v}}\left[\dfrac{\mu_1(\bar{\lambda})F_z}{m}\left((1-\bar{\lambda})\left(1+K\dfrac{r}{Jg}(1-\alpha)\right)+\dfrac{mr^2}{J}\right)+K\alpha\dfrac{r}{J}\right]}$$



Noise sensitivity



Slip Vs. MSD (α = 0.9)



Slip-control: linear stability analysis

$[m=225, J=1, r=0.28]$

$$G_\lambda(s) = \frac{\left[\dfrac{r}{vJ}\right]}{s + \left[\dfrac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda}) + \dfrac{mr^2}{J}\right)\right]} \rightarrow$$

$$G_\lambda(s) = \frac{\dfrac{0.28}{\bar{v}}}{s + \dfrac{1}{\bar{v}}\left[\mu_1(\bar{\lambda})9.8\left((1-\bar{\lambda}) + 17.6\right)\right]}$$

Slip-control: linear stability analysis

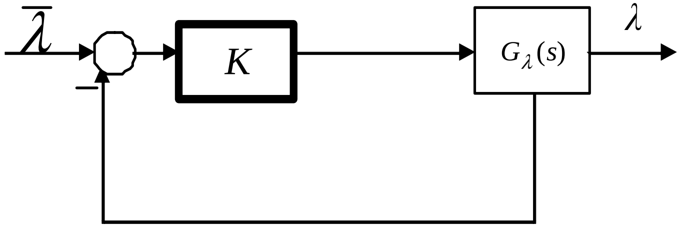
$$1000\left[\frac{\frac{0.28}{30}}{s+\frac{60}{30}}\right]$$

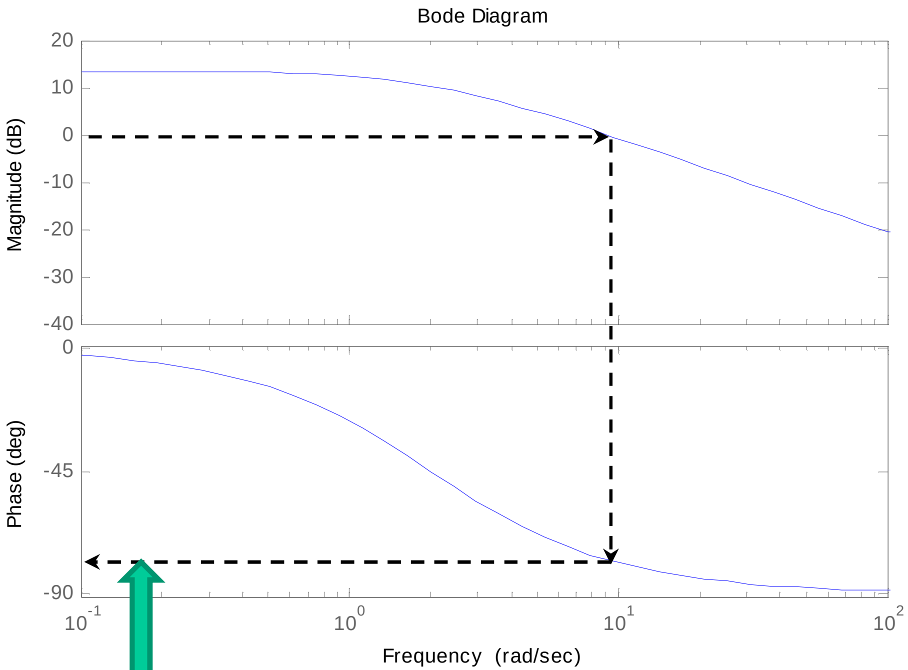
No unstable open-loop pole

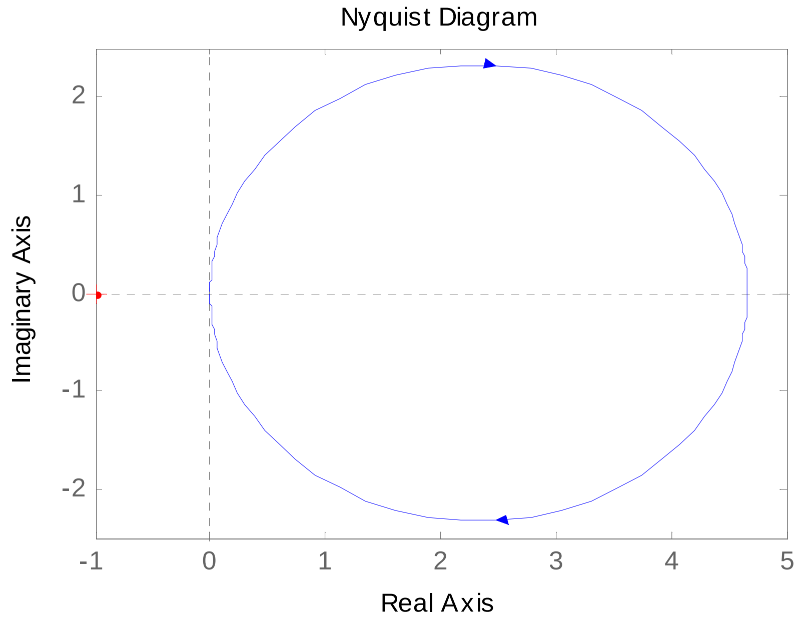
No (-1,0) (counter-clock-wise) encircling

Asymptotic stability

Bode criterion is applicable





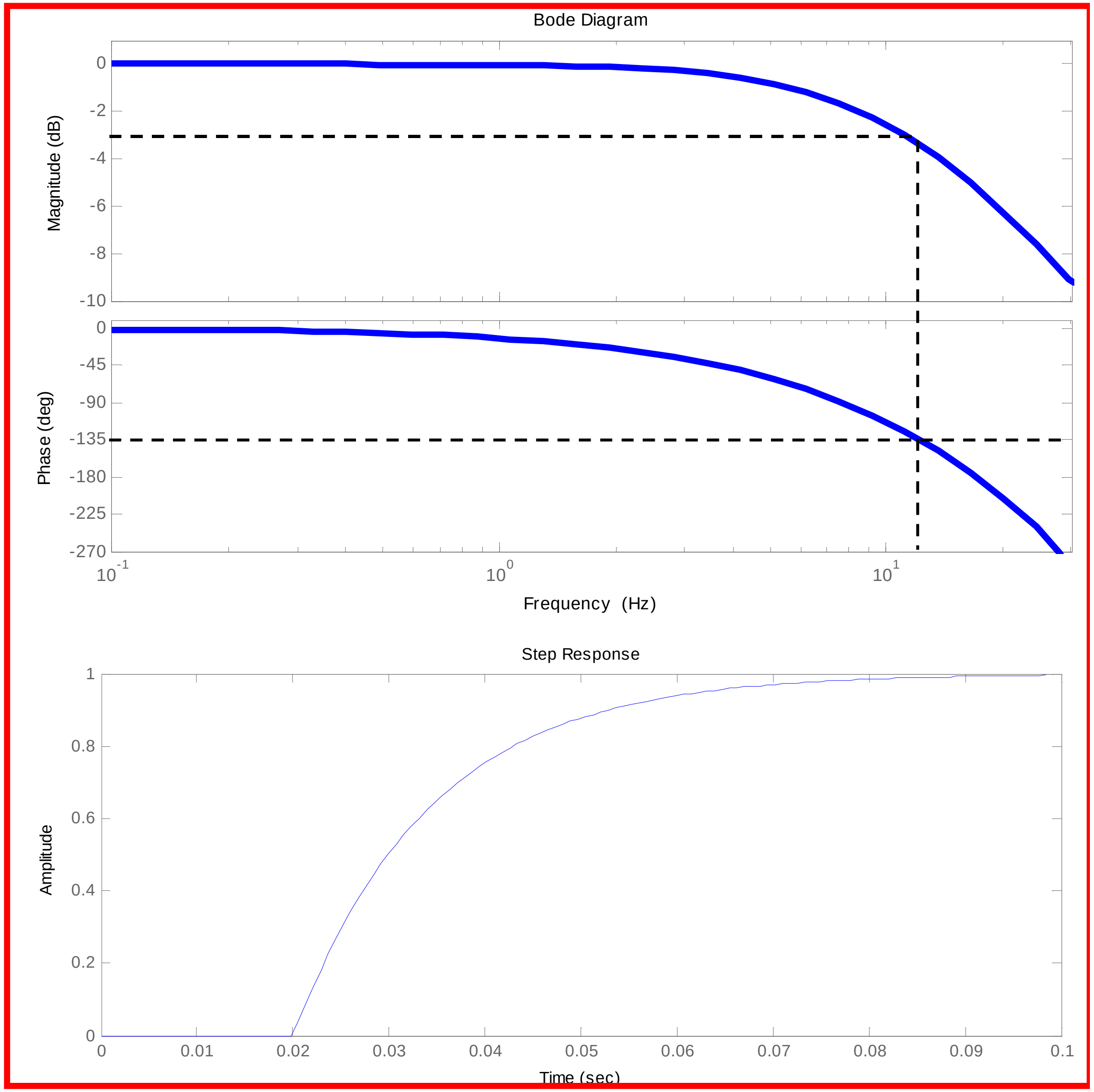


Phase margin = distance to “-180°”

Actuator dynamics

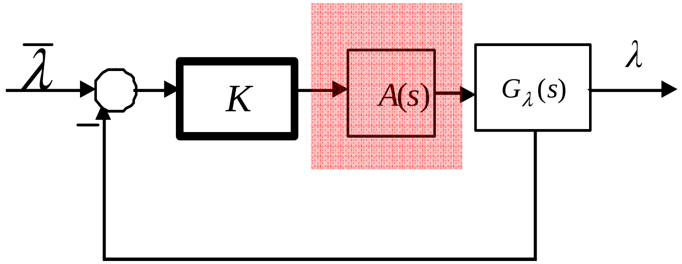
Added dynamics (example)

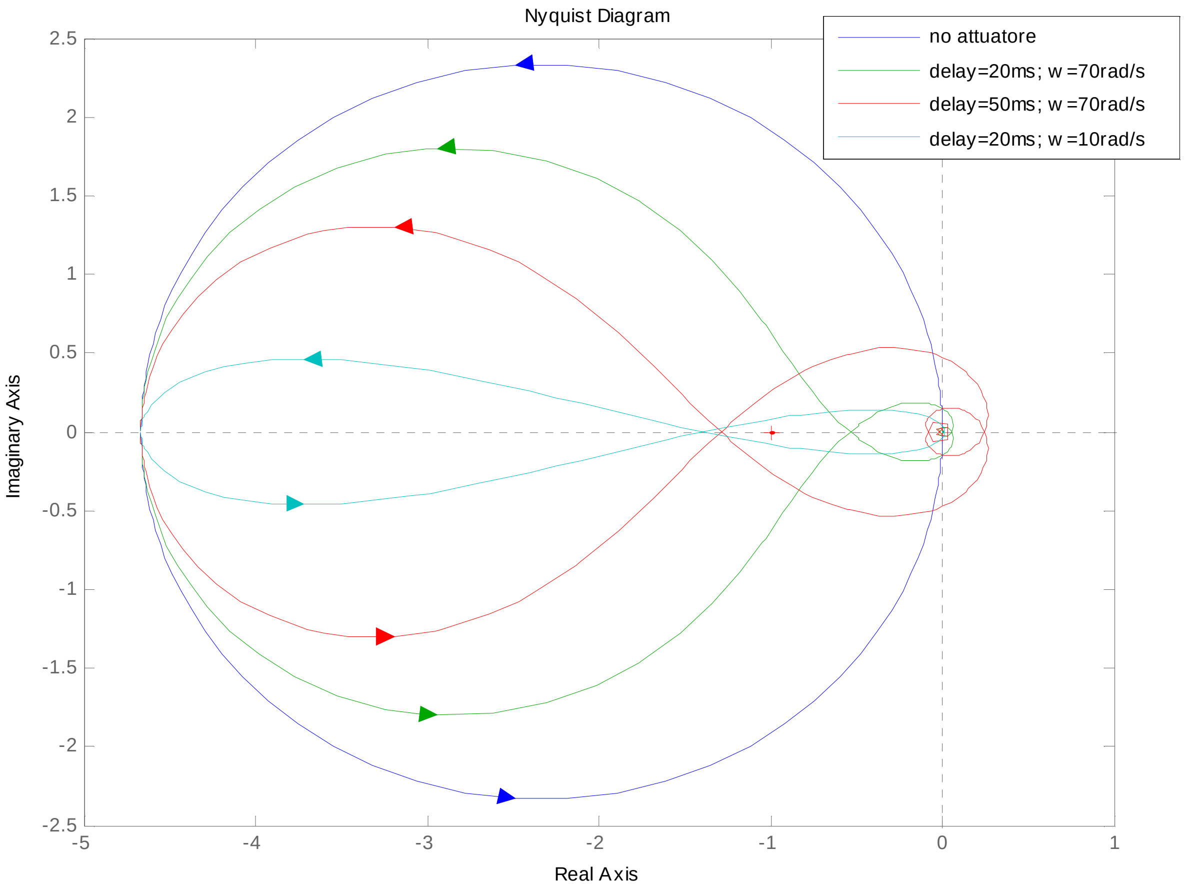
$$F_{caliper}(s) = \frac{70}{s+70} e^{-(0.02)s}$$



Example - effect of the actuator dynamics

$$1000\left[\frac{\frac{0.28}{10}}{s-\frac{60}{10}}\right]\left[\frac{70}{s+70}\right]\left[e^{-(0.02)s}\right]$$





Slip-control: example of a PID design

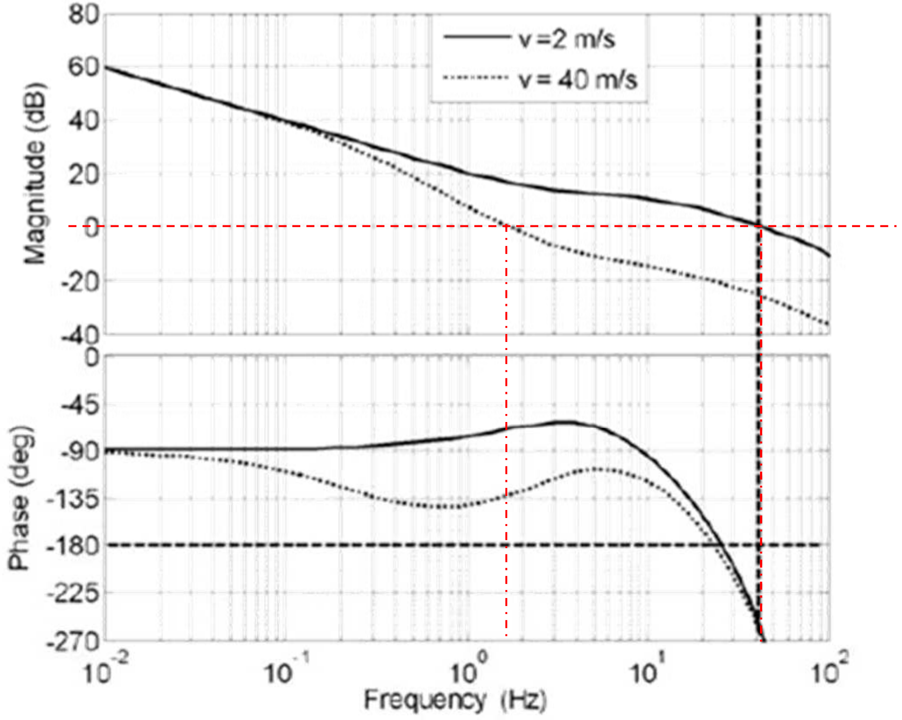


Figure 3.8 Magnitude and phase Bode diagrams of the frequency response associated with the loop transfer function $L_{2\lambda}(s)$ for $\overline{v} = 40$ m/s (*dotted line*) and $\overline{v} = 2$ m/s (*solid line*)

$$L_{2\lambda}(s) = R_\lambda(s)D^{\mathrm{st}}(s),$$

$$D^{st}(s) = G_{\mathrm{caliper}}(s)G_\lambda^{\mathrm{st}}(s)$$

$$G_\lambda^{\mathrm{st}}(s) = \frac{0.3/\overline{v}}{s + 60/\overline{v}},$$

$$R_\lambda(s) = 12000\frac{(1+\frac{1}{20}s)^2}{s(1+\frac{1}{500}s)}.$$

Cut-off frequency: between
2 and 40Hz about

$$G_{\mathrm{caliper}}(s) = \frac{\omega_{\mathrm{act}}}{s+\omega_{\mathrm{act}}}e^{-s\tau}, \qquad (1.4)$$

At 2m/s becomes unstable

with $\omega_{\mathrm{act}} = 70$ rad/s and $\tau = 10$ ms.

Recall: “Ideal” PID

Controllers **PID** $\rightarrow$ action **P**roportional,**In**tegral, **D**erivative.

$$e(t) \longrightarrow \boxed{\text{PID}} \longrightarrow u(t)$$

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d \frac{de(t)}{dt}$$

Coeff. Proportional

Coeff. Integral Action

Coeff. Derivative Action

$$u(t) = K_p \left[e(t) + \frac{1}{T_i} \int_0^t e(\tau)d\tau + T_d \dot{e}(t) \right]$$

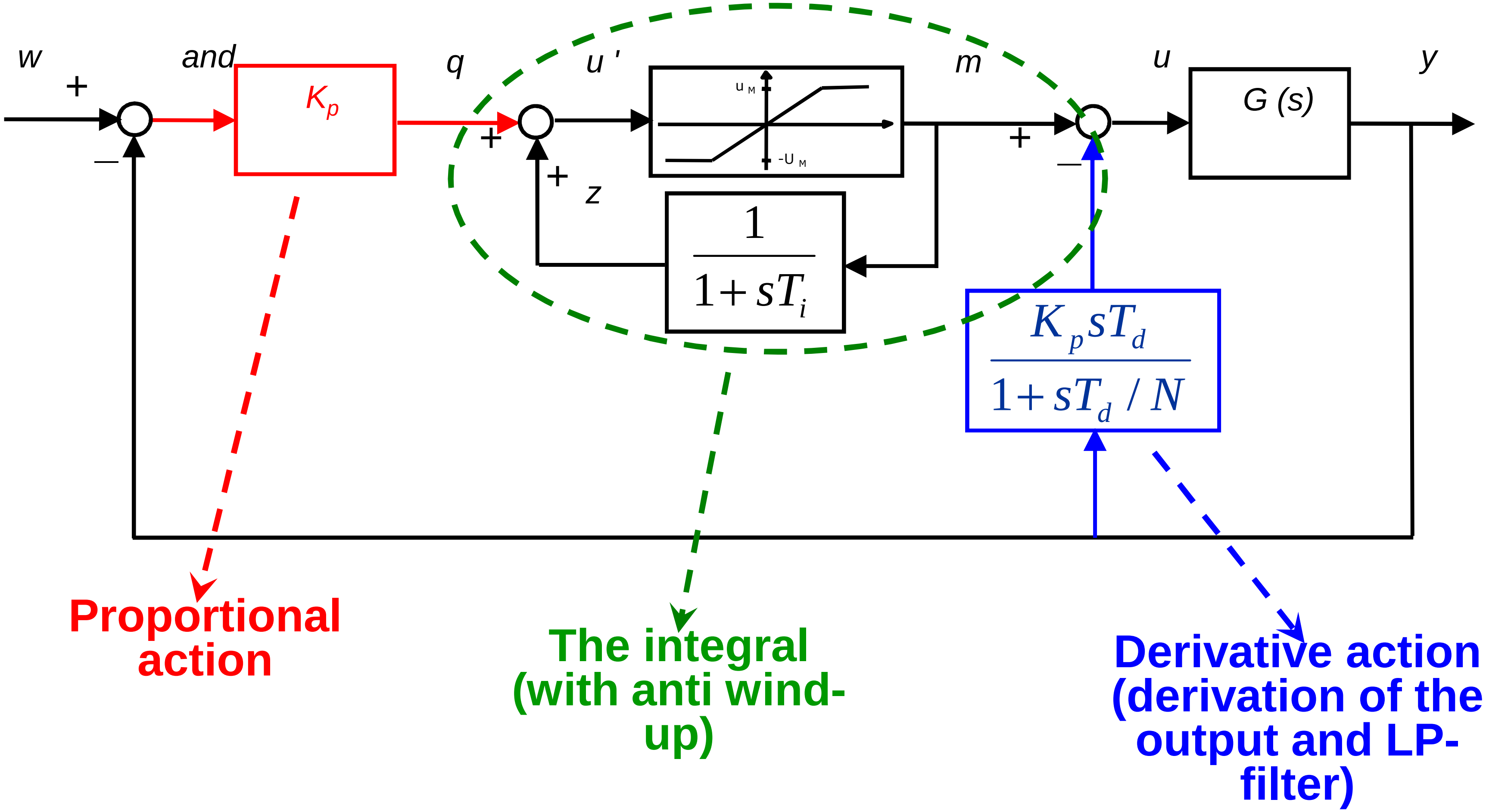
Integral action time

Derivative action time

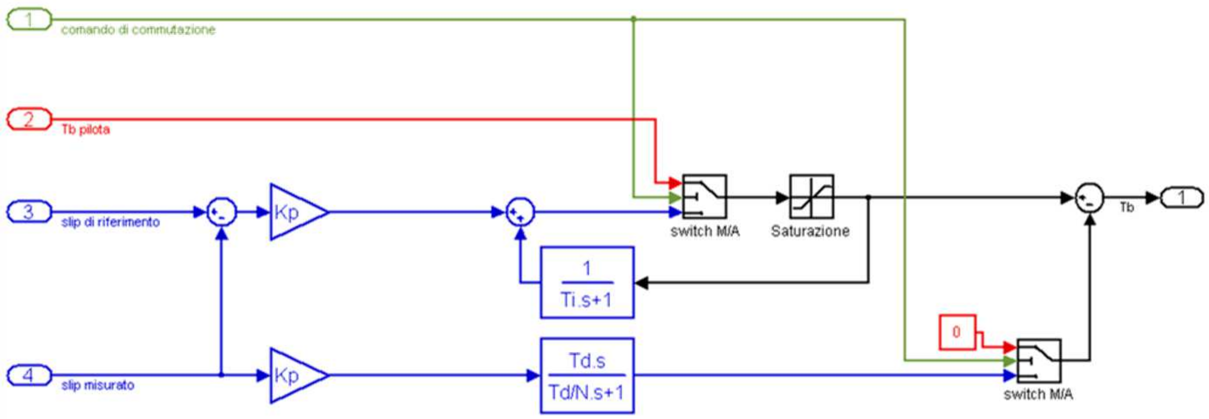
$$T_i = \frac{K_p}{K_i}$$

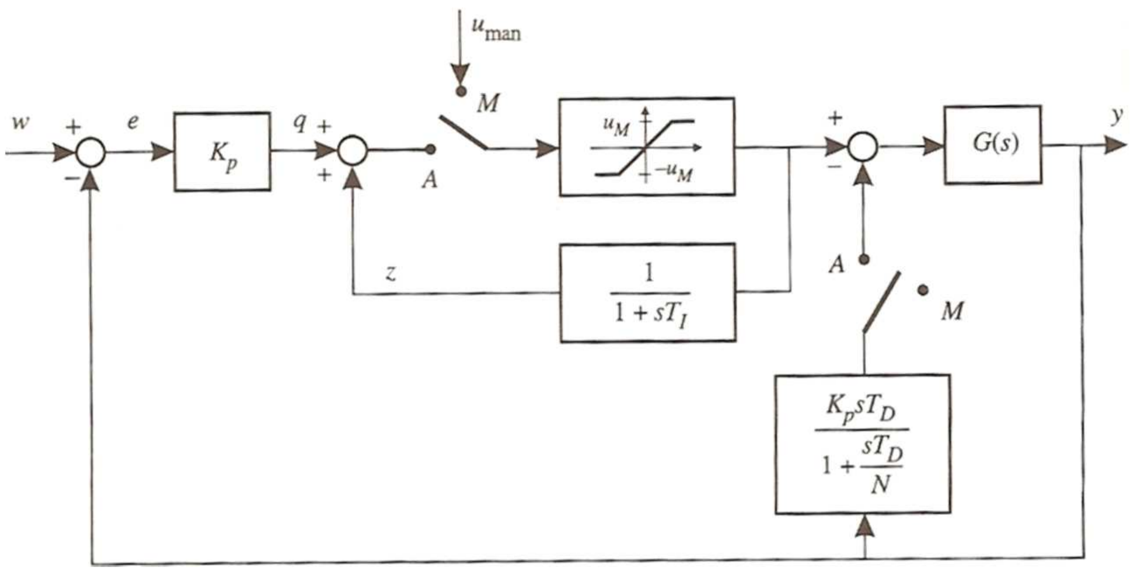
$$T_d = \frac{K_d}{K_p}$$

Recall: "real" PID



Logic activation-deactivation on a PID



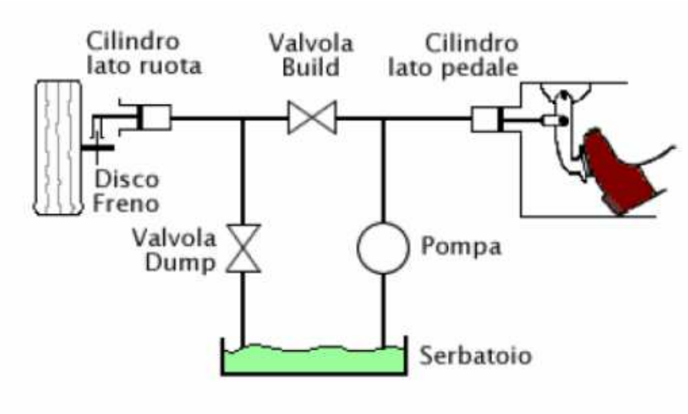


activation and deactivation variable(s)?

Slip-control algorithms (ABS) with three-states-actuators

Actuator

HAB: Hydraulic Actuated Brakes (increase, hold, decrease)



Constraints:

1. Rate limiter $dT_b/dT=u$
2. Control action: $u \in \{k, 0, -k\}$

Control actions:

Open Build - Close dump → Increase pressure (u = k)

Close Build - Open dump → decrease pressure (u =-k)

Build Close - Close dump → hold pressure (u = 0)

Model (recall)

$$\begin{cases} \dot{\lambda} = -\frac{1}{v}\left(\frac{(1-\lambda)}{m} + \frac{r^2}{J}\right) F_z \mu(\lambda) + \frac{r}{vJ} T_b \\ m\dot{v} = -F_z \mu(\lambda) \end{cases}$$

Hp: v is a parameter ("slowly varying")

$$\dot{\lambda} = -\frac{1-\lambda}{\omega r}\left(\frac{(1-\lambda)}{m} + \frac{r^2}{J}\right) F_z \mu(\lambda) + \frac{r}{J}\frac{1-\lambda}{\omega r} T_b$$

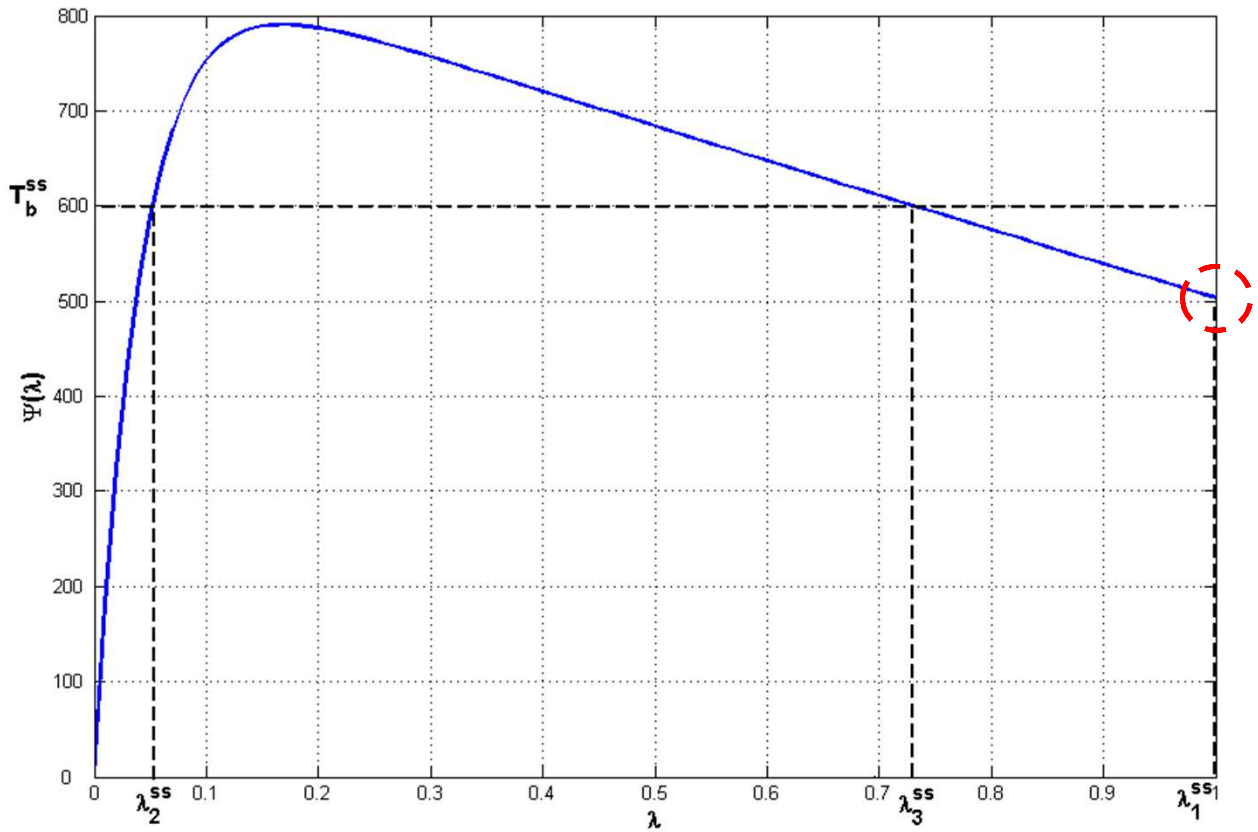
Equilibria (open loop)

$$\dot{\lambda} = -\frac{1-\lambda}{\omega r}\left(\frac{(1-\lambda)}{m} + \frac{r^2}{J}\right) F_z \mu(\lambda) + \frac{r}{J}\frac{1-\lambda}{\omega r} T_b$$

$$\boxed{\begin{aligned} &\lambda = \lambda^{ss} \\ &T_b = T_b^{ss} = \Psi(\lambda) = F_z \mu(\lambda)\left(r + \frac{J}{rm}(1-\lambda)\right) \end{aligned}}$$

Equilibria (open loop)

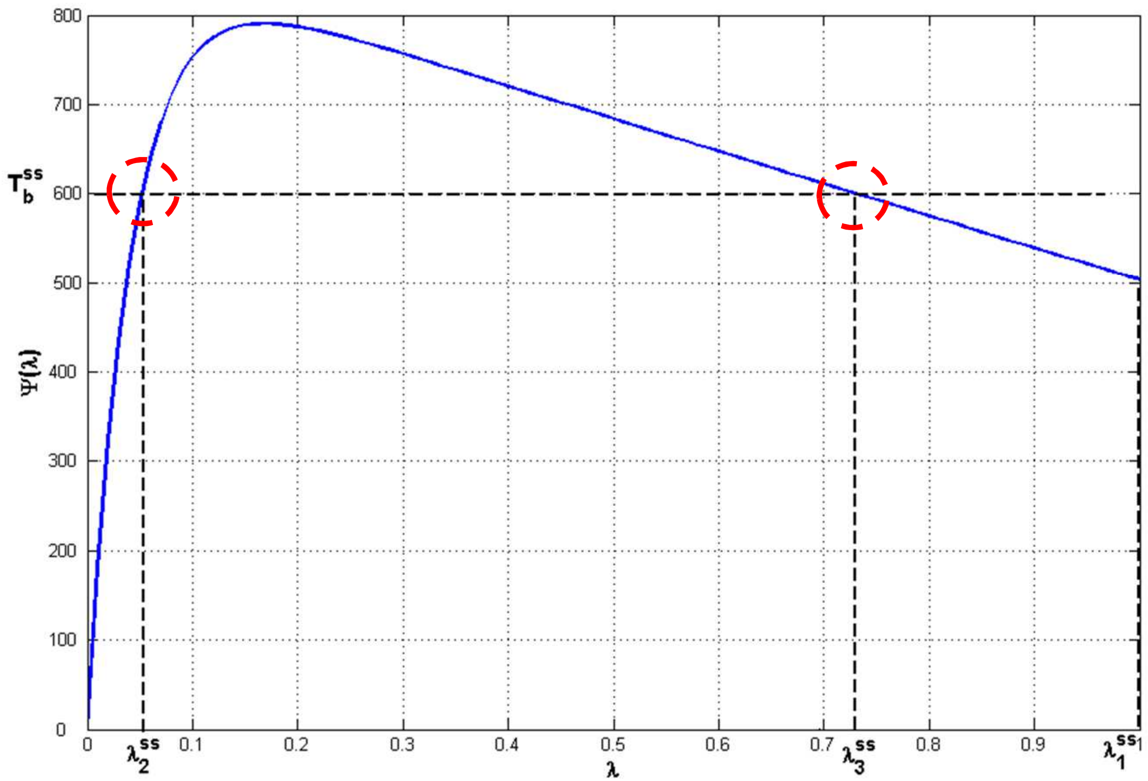
$$T_b^{ss} > \max_{\lambda} \Psi(\lambda) \rightarrow \lambda_1^{ss} = 1$$



Equilibria (open loop)

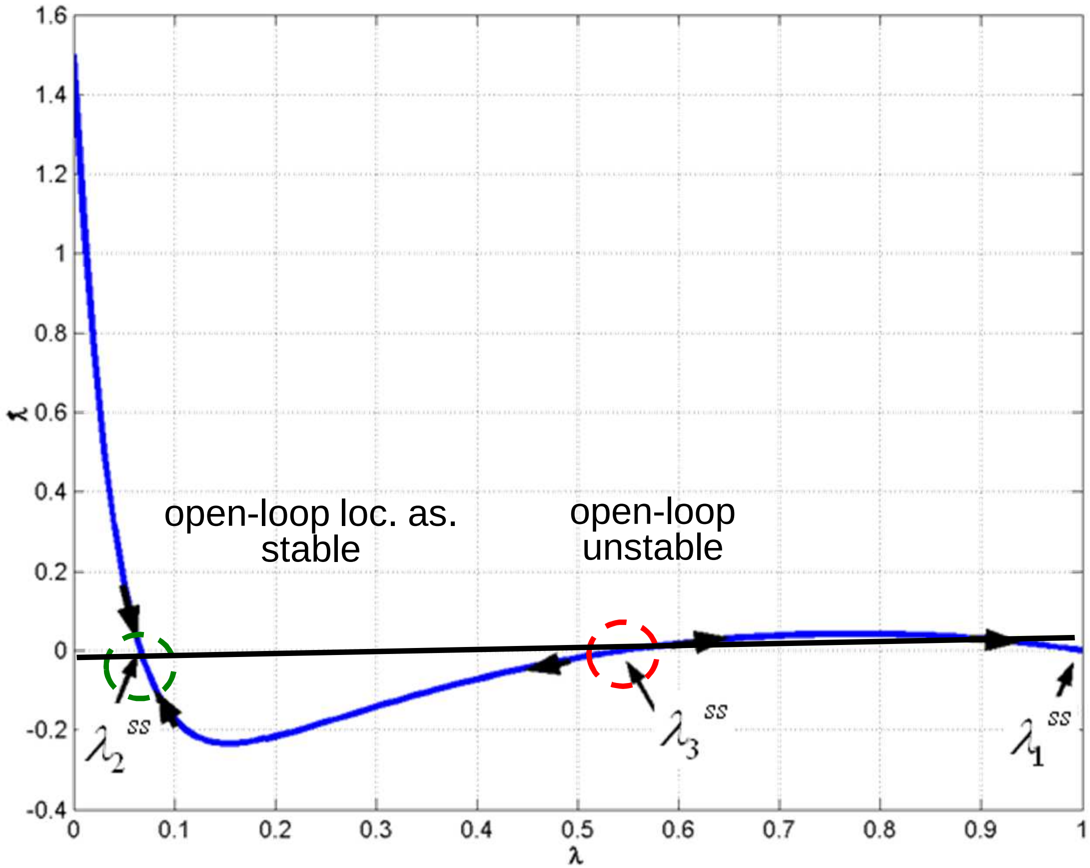
$$T_b^{ss} \leq \max_{\lambda} \Psi(\lambda)$$

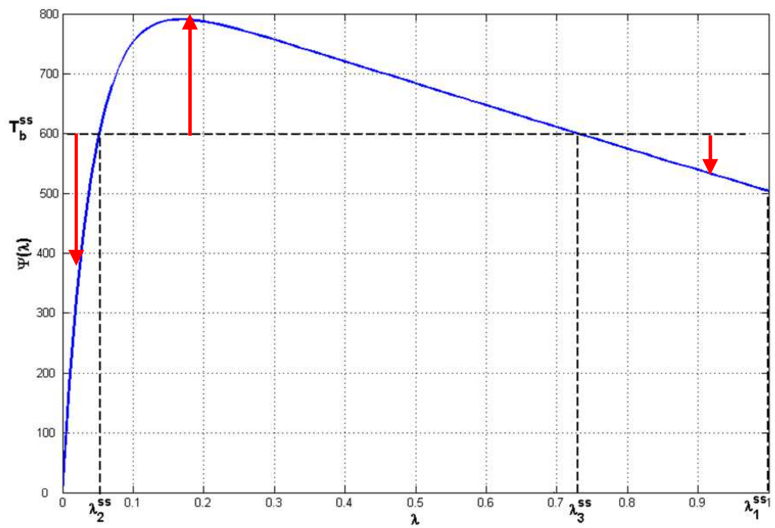
$$\lambda_2^{ss} = \bar{\lambda}_2, \qquad \lambda_3^{ss} = \bar{\lambda}_3$$



Phase diagrams (open-loop)

$$\dot{\lambda} = -\frac{(1-\lambda)}{J\omega}(\Psi(\lambda) - T_b)$$





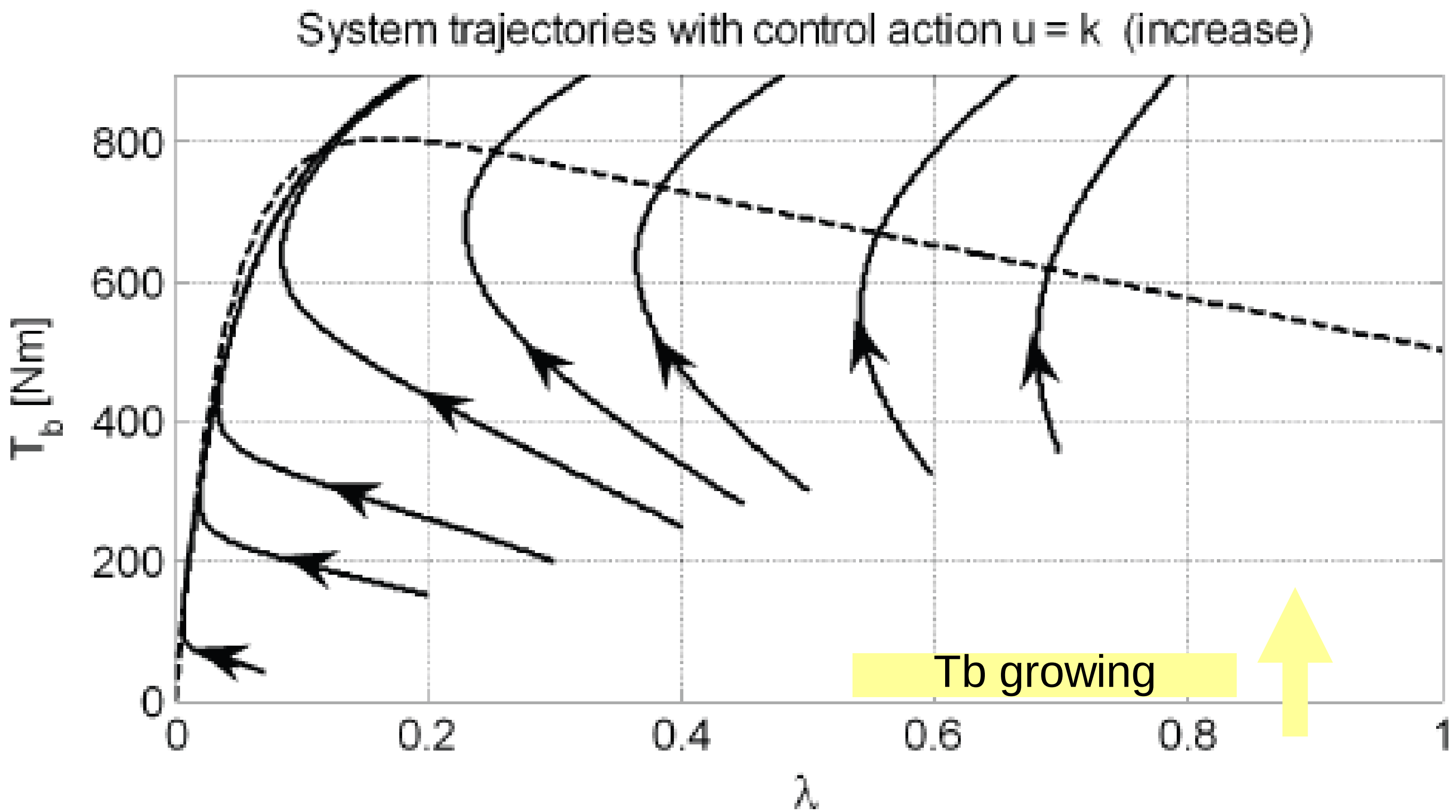
Analysis of closed loop

$$\begin{cases} \dot{\lambda} = -\frac{1-\lambda}{\omega r}\left(\frac{(1-\lambda)}{m} + \frac{r^2}{J}\right) F_z \mu(\lambda) + \frac{r}{J}\frac{1-\lambda}{\omega r} T_b \\ \dot{T}_b = u \end{cases}$$

We analyze the phase diagrams: $u \in \{k, 0, -k\}$

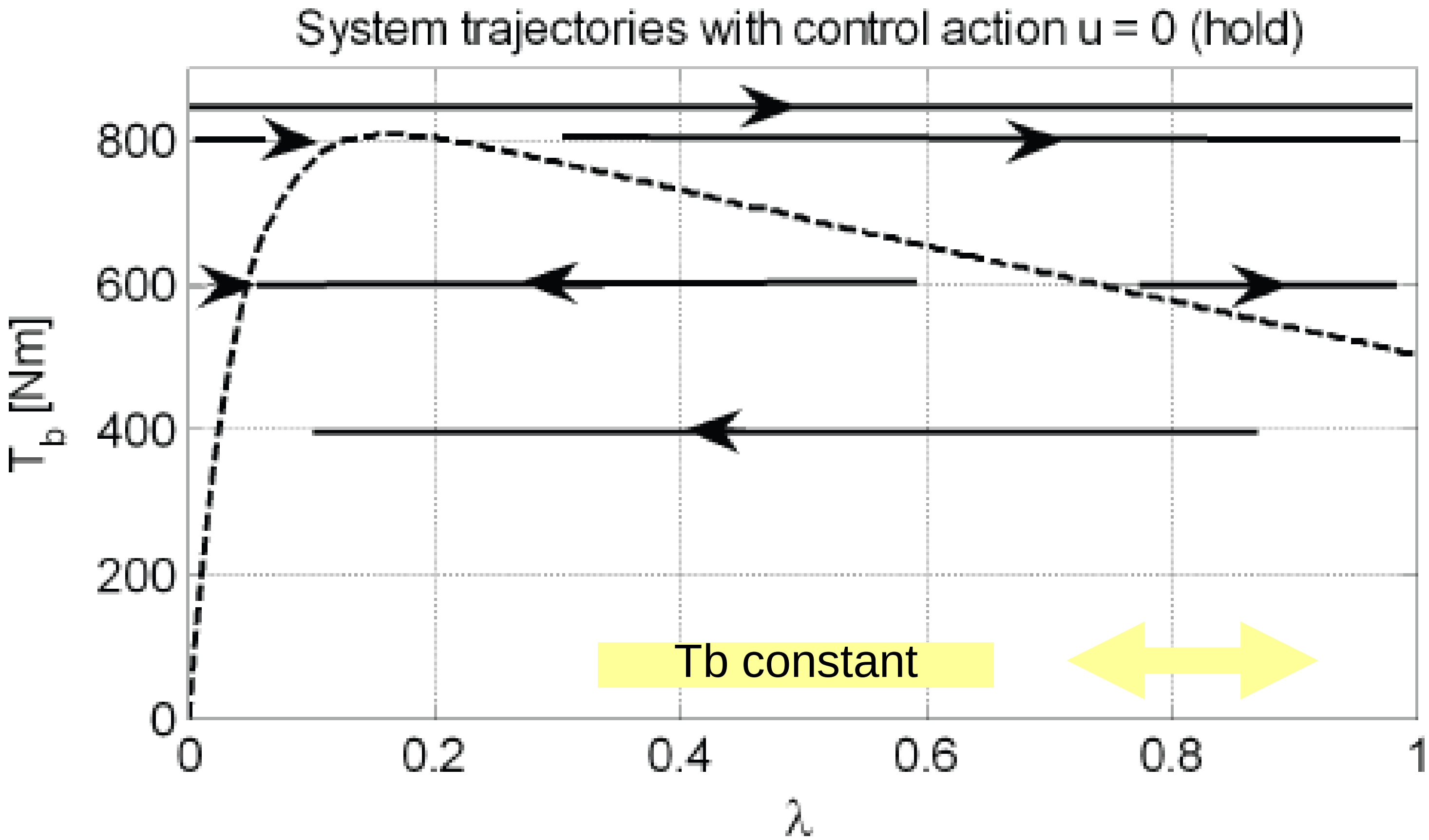
Increase: *u = k*

$$\dot{\lambda} = -\frac{(1-\lambda)}{J\omega}(\Psi(\lambda) - T_b)$$



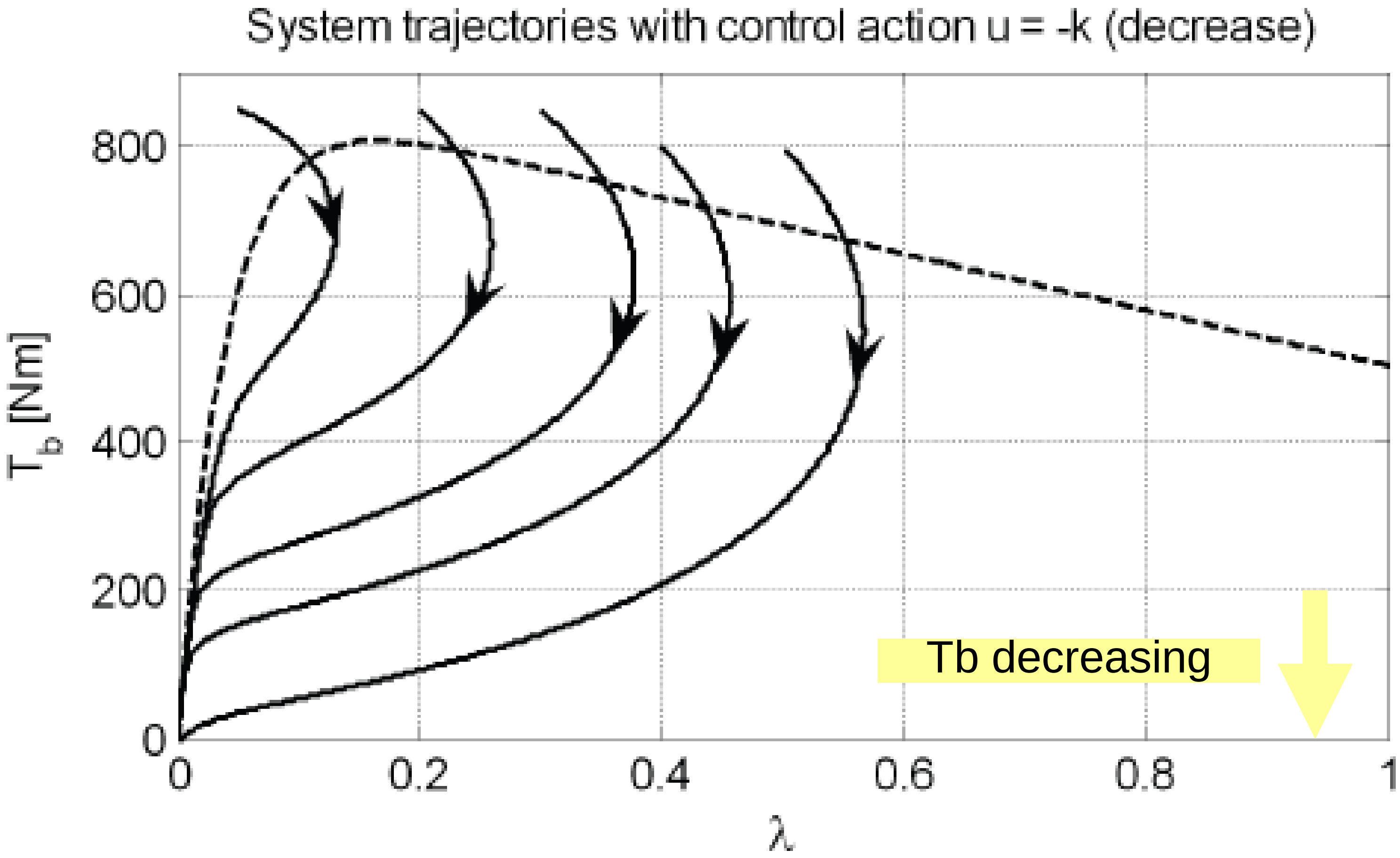
Hold: *u = 0*

$$\dot{\lambda} = -\frac{(1-\lambda)}{J\omega}\left(\Psi(\lambda) - T_b\right)$$



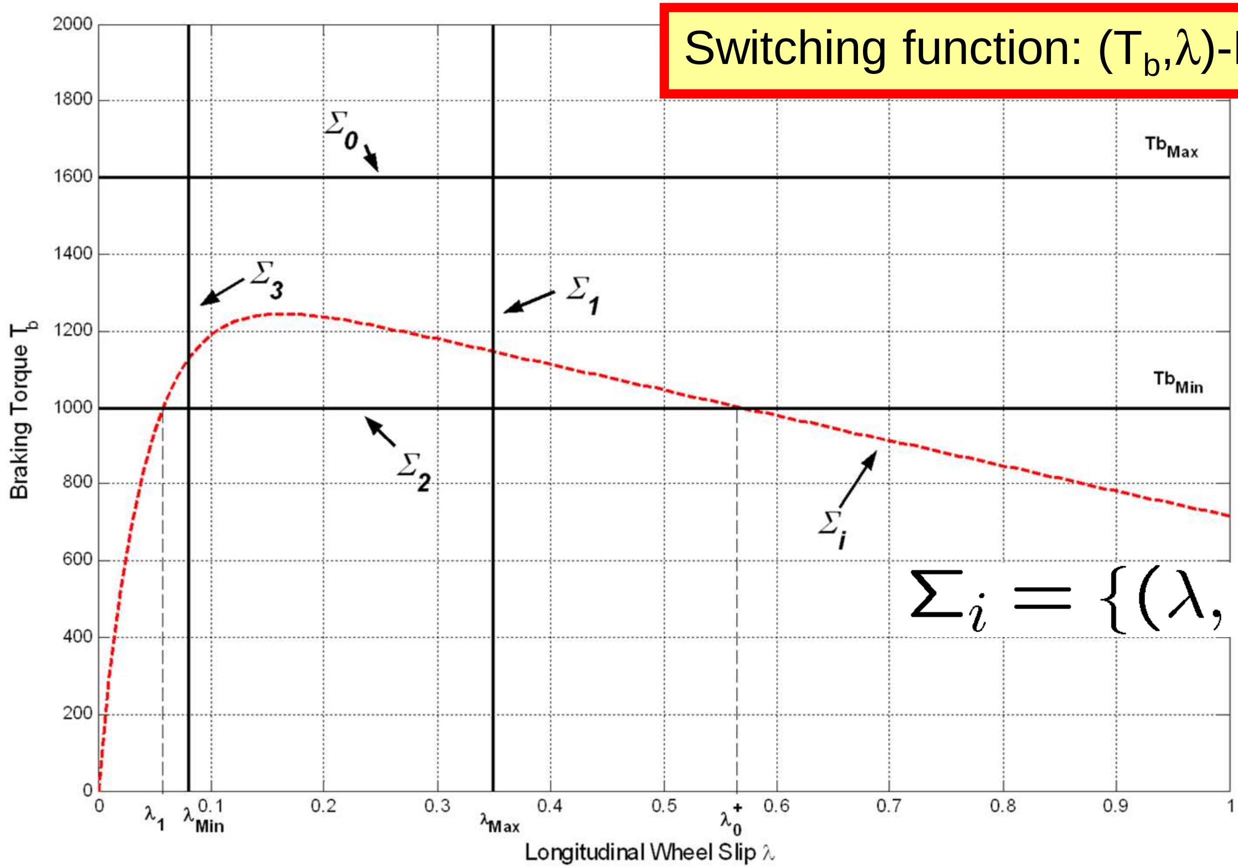
Decrease: *u =-k*

$$\dot{\lambda} = -\frac{(1-\lambda)}{J\omega}\left(\Psi(\lambda) - T_b\right)$$



Switching Control Logic

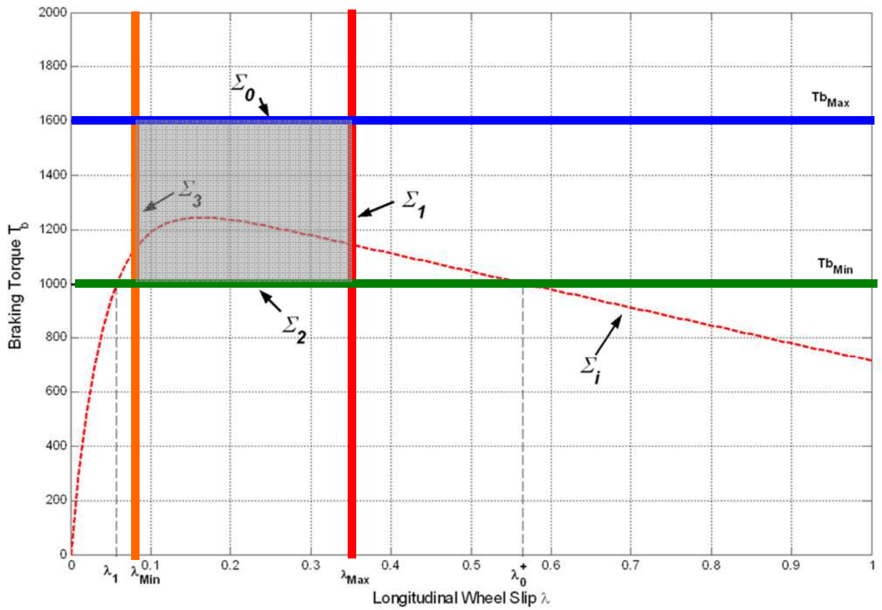
Switching function: (T_b,λ)-Box

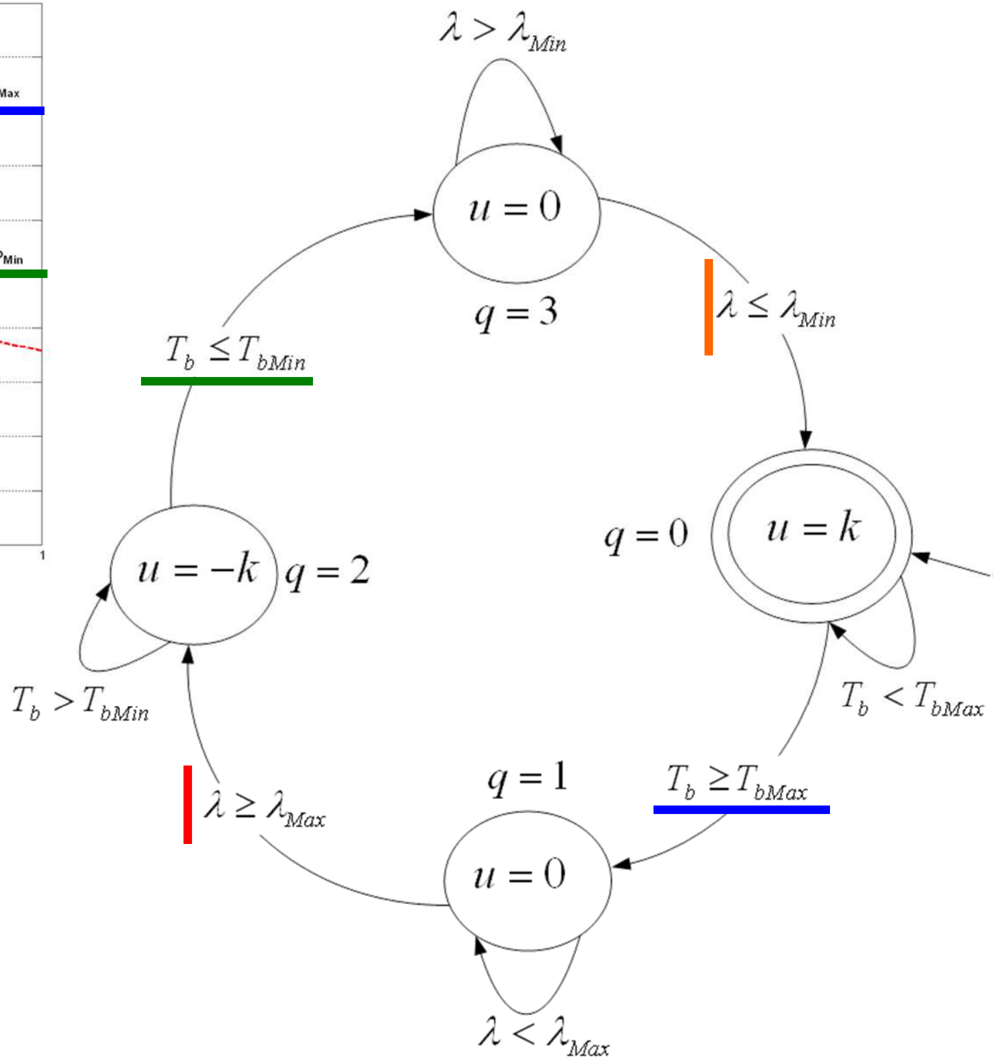


$$\Sigma_i = \{(\lambda, T_b) \; : \; T_b = \Psi(\lambda)\}$$

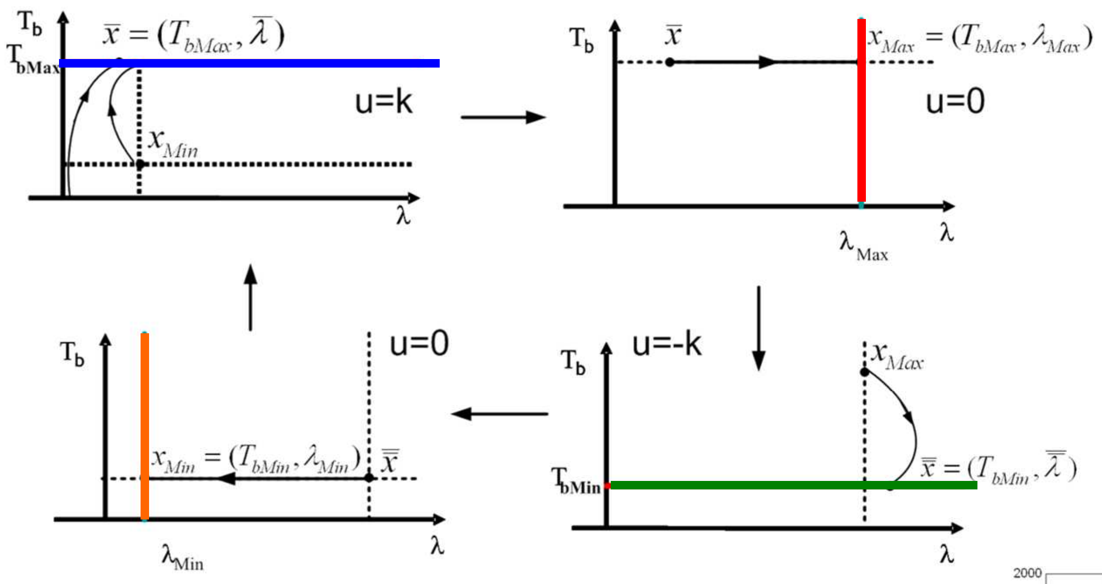
$$\Sigma_0 = \{(\lambda, T_b) \; : \; H_0(\lambda, T_b) := T_b - T_{bMax} = 0\},$$
$$\Sigma_1 = \{(\lambda, T_b) \; : \; H_1(\lambda, T_b) := \lambda - \lambda_{Max} = 0\},$$
$$\Sigma_2 = \{(\lambda, T_b) \; : \; H_2(\lambda, T_b) := T_b - T_{bMin} = 0\},$$
$$\Sigma_3 = \{(\lambda, T_b) \; : \; H_3(\lambda, T_b) := \lambda - \lambda_{Min} = 0\}.$$

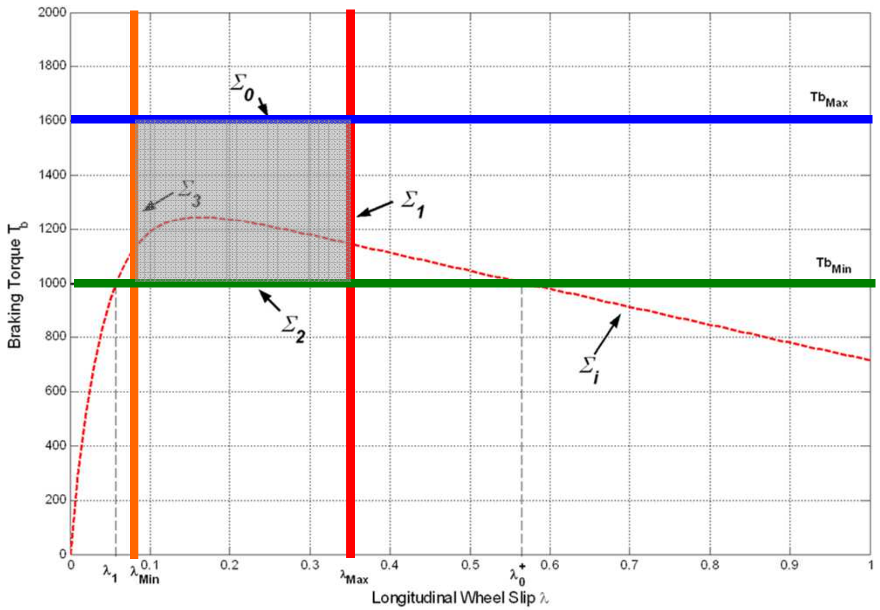
Switching Control Logic



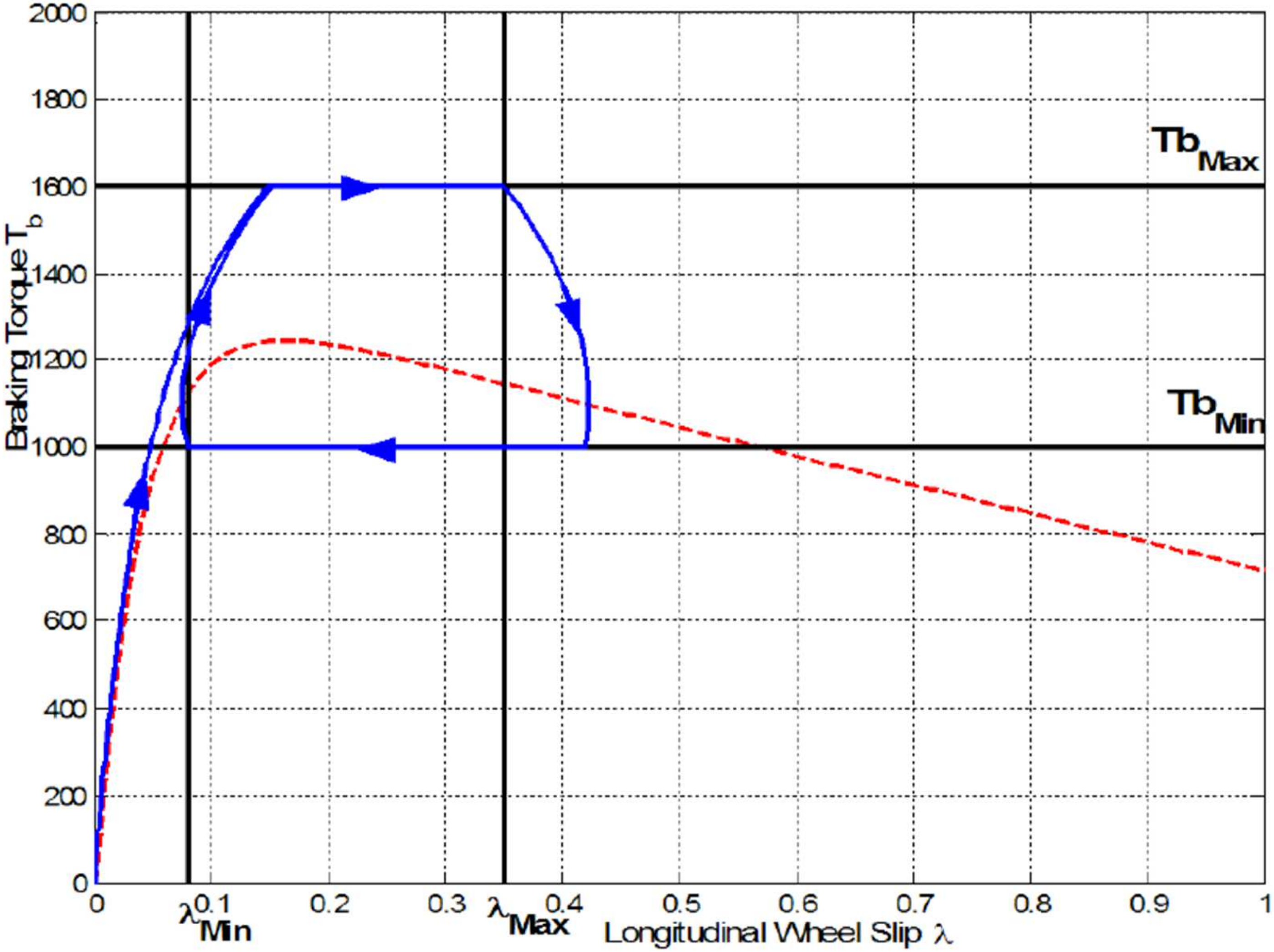


Limit-cycle: analysis

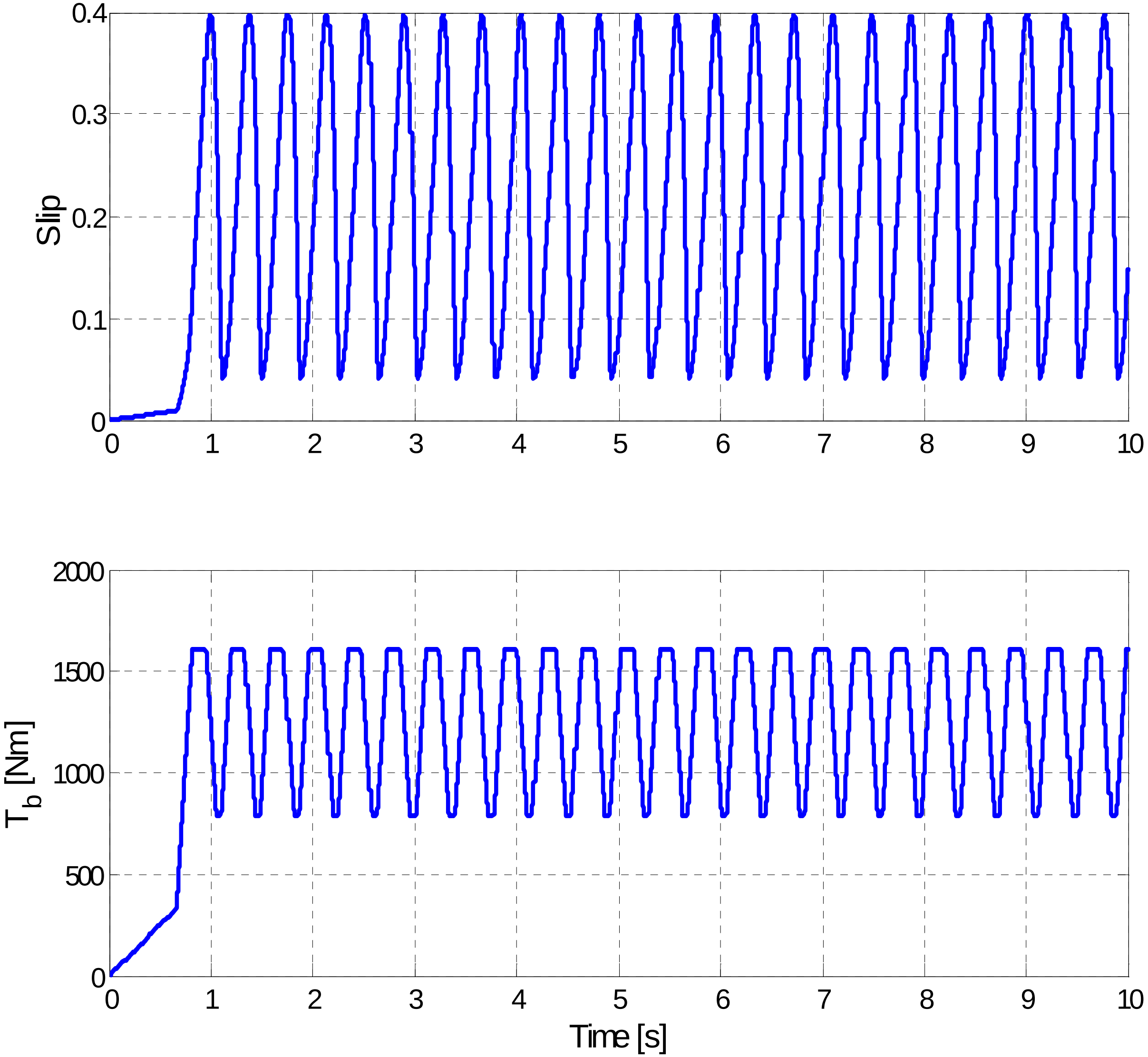




Limit-cycle: analysis



Limit-cycle: analysis



Limit-cycle: view in the hybrid space

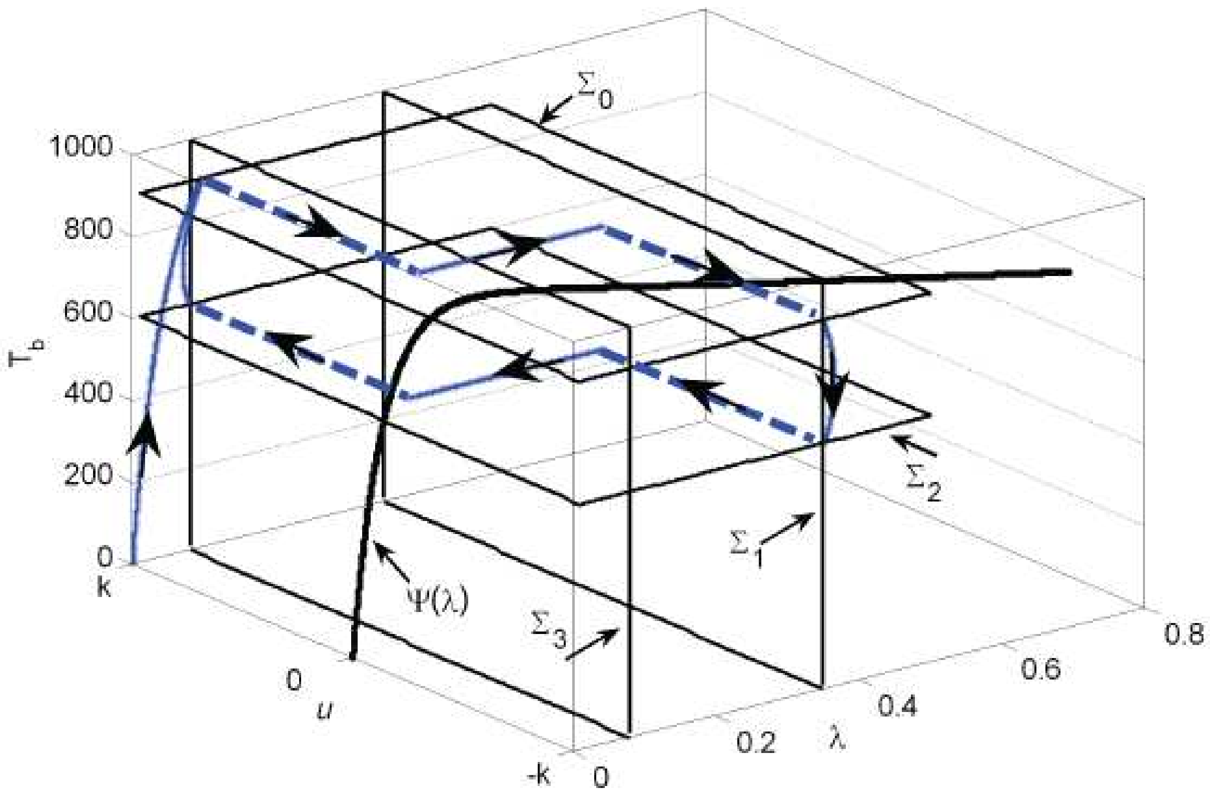
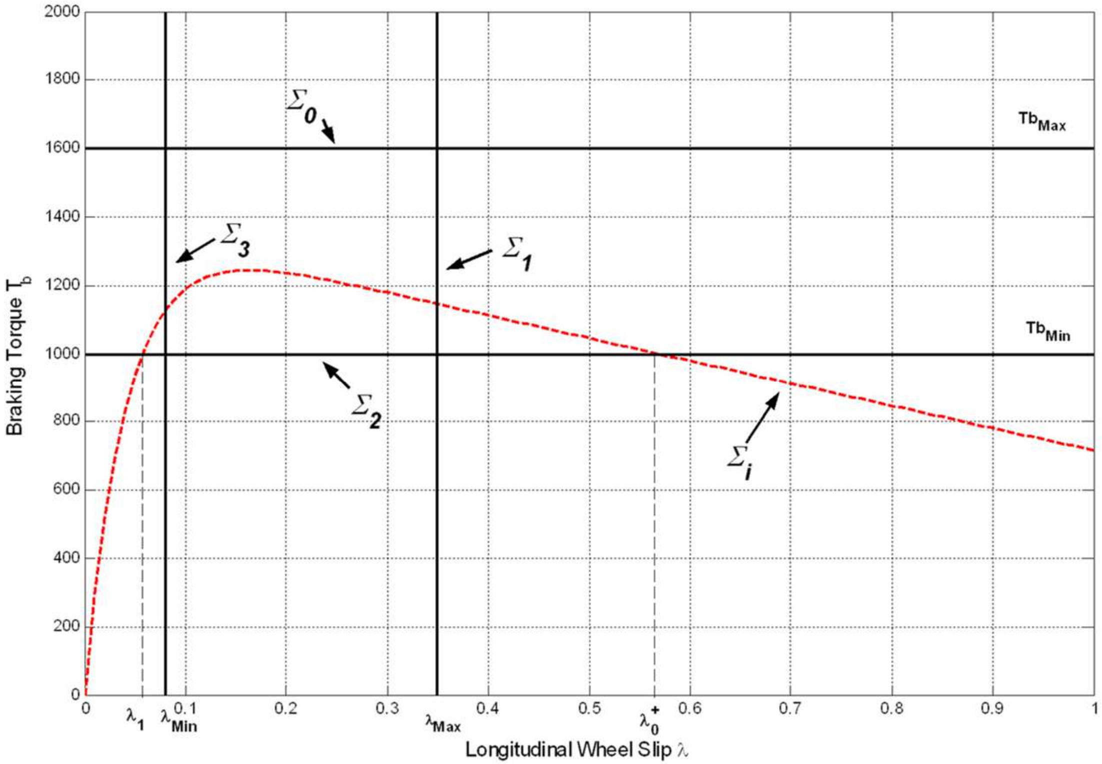


Figure 4.3 Simulated closed-loop trajectory of system (4.1) with the given control logic in the *hybrid* state space (T_b, λ, u)

Existence of limit cycles: necessary conditions

$$\Sigma_0 \cap \Sigma_i = \emptyset$$

$$\Sigma_2 \cap \Sigma_i \neq \emptyset$$



if $\Sigma_2 \cap \Sigma_i = \{(T_{bMin}, \lambda_1), (T_{bMin}, \lambda_0^+)\}, \; \lambda_1 < \lambda_0^+$

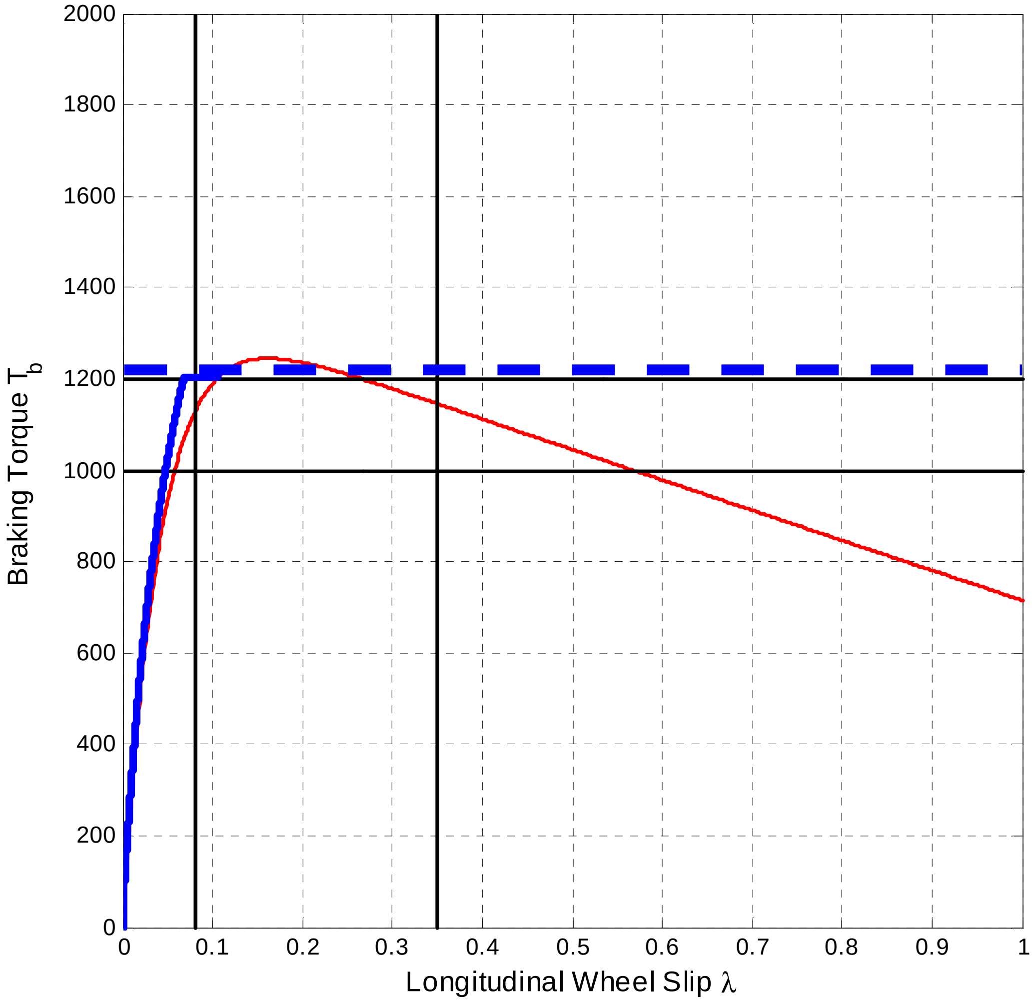
then the intersection between the system trajectory

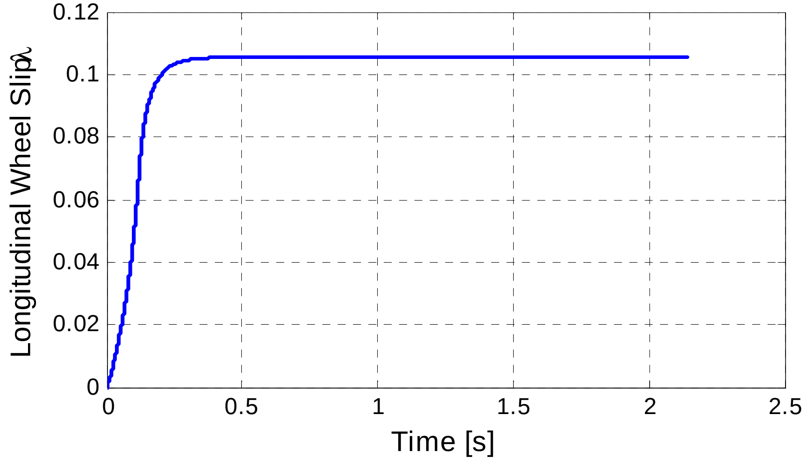
$$\Phi(t, [\lambda_{\mathrm{Max}}, T_{b\mathrm{Max}}; q = 2])$$

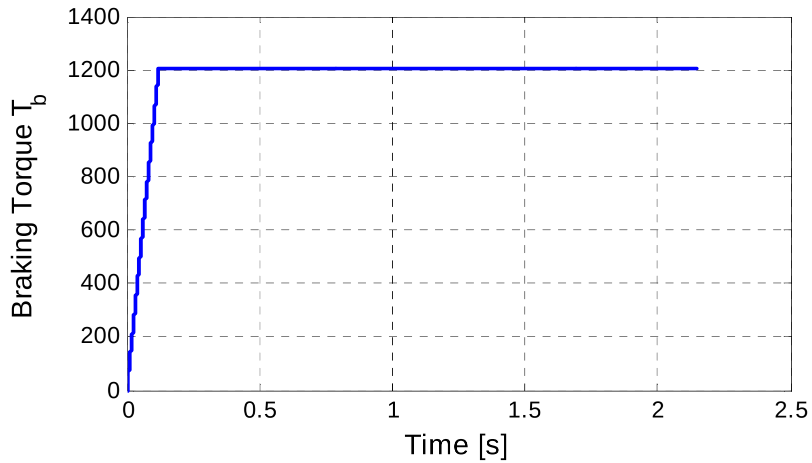
must occur at a point $(T_{b\mathrm{Min}}, \overline{\overline{\lambda}})$ such that $\overline{\overline{\lambda}} < \lambda_0^+$

Limit-cycle: analysis

First condition violated: NOT a limit cycle but a steady-state condition

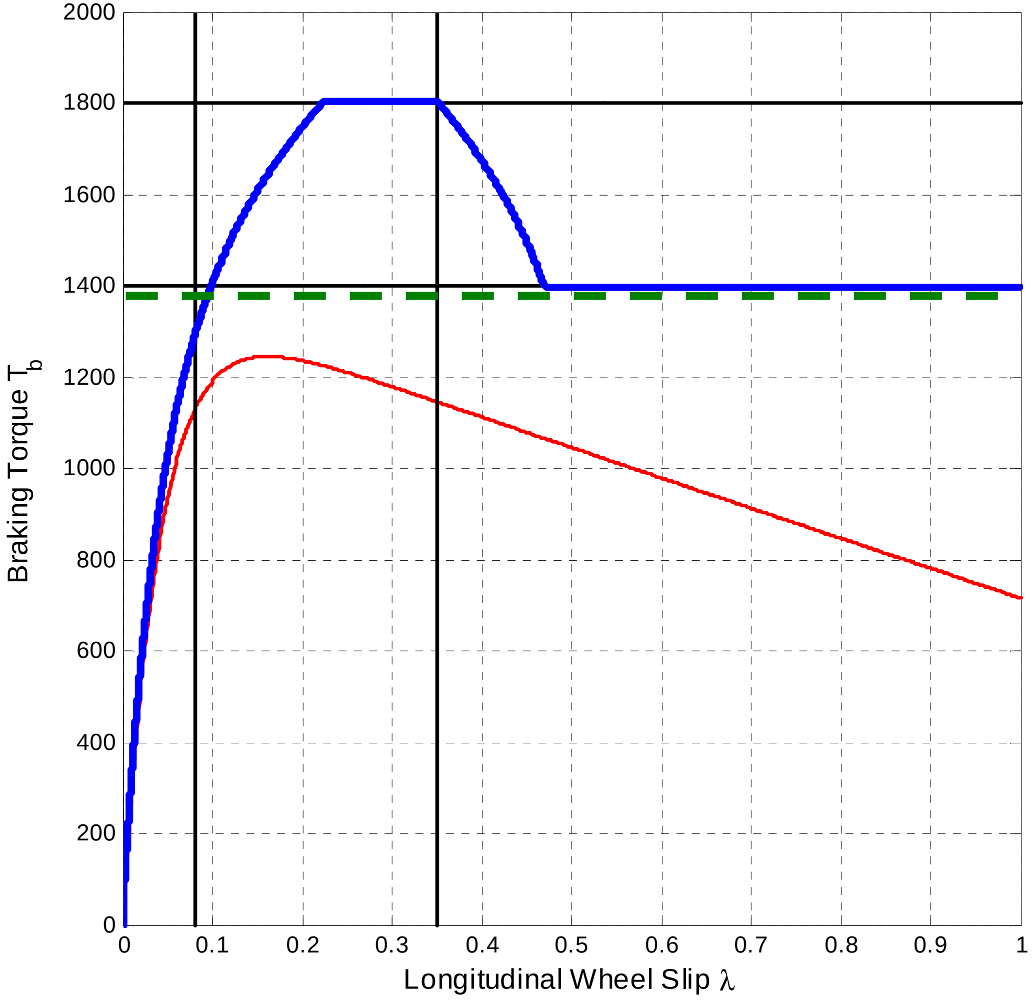


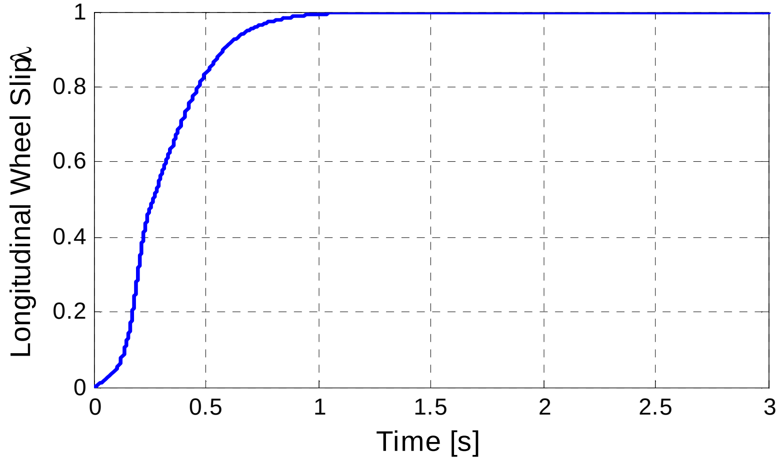


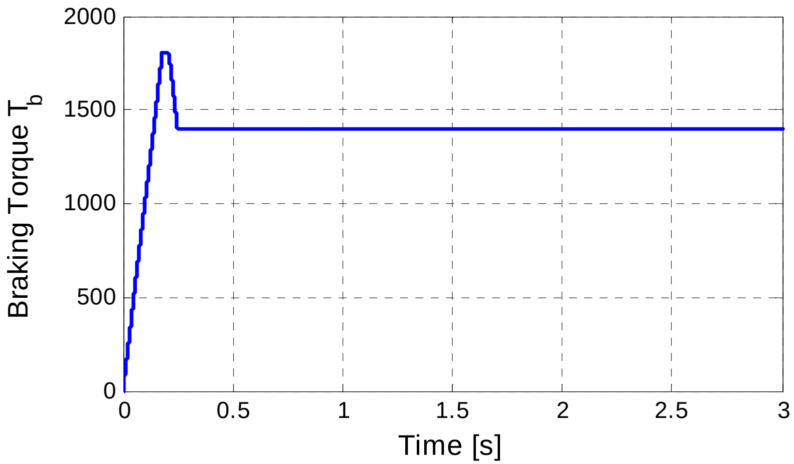


Limit-cycle: analysis

The second condition is violated: NOT a limit cycle but wheel locks

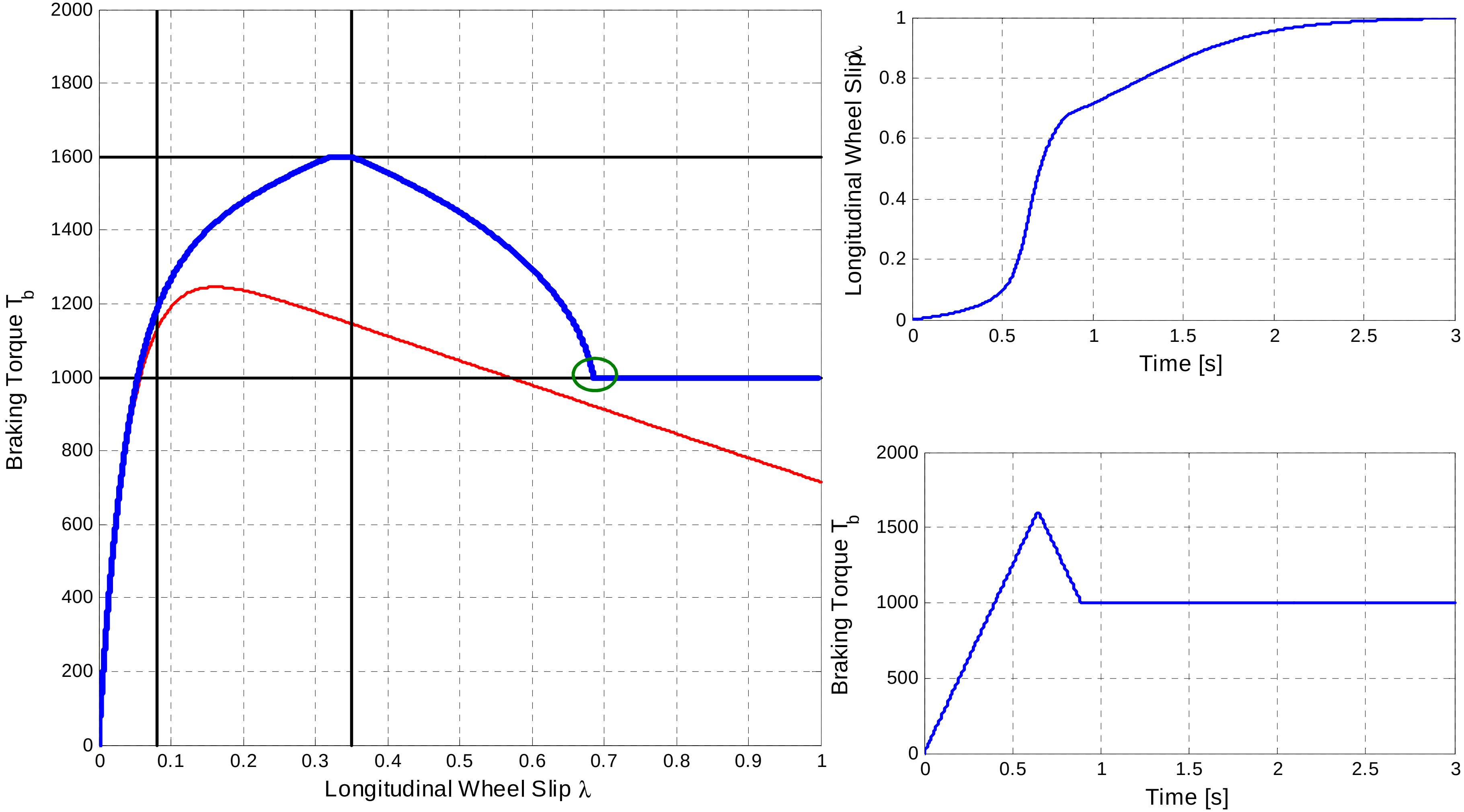






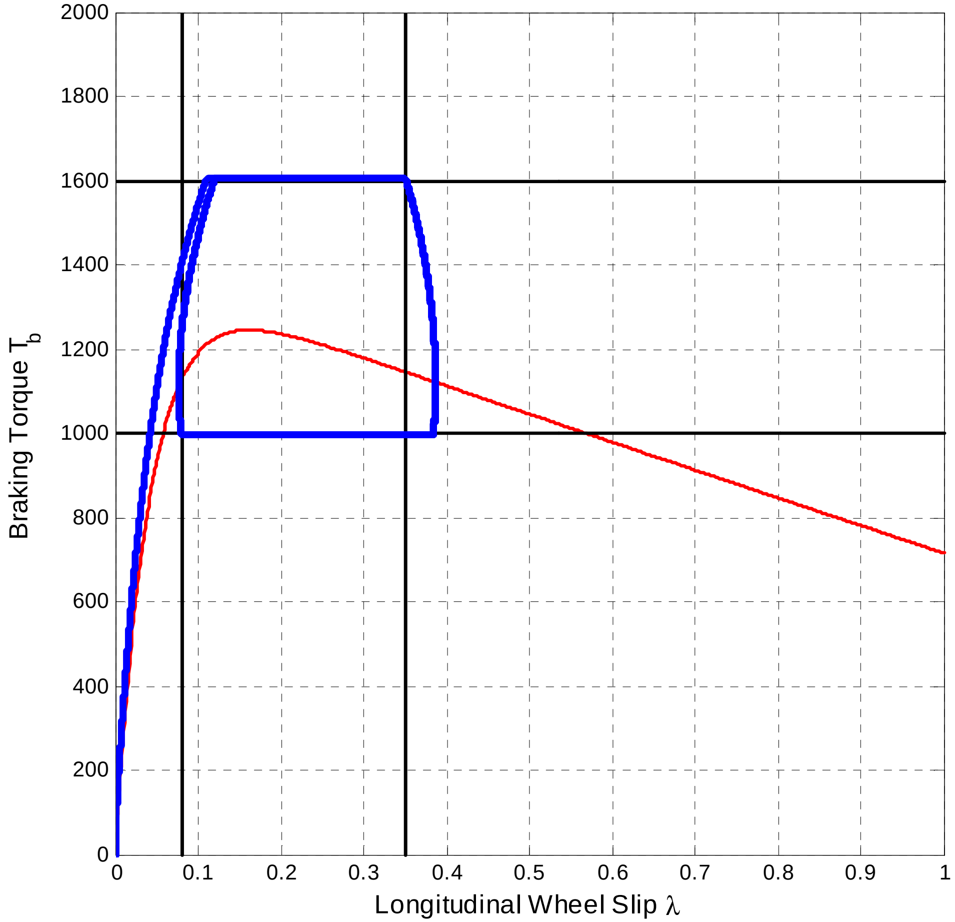
Limit-cycle: analysis

Third condition violated: NOT a limit cycle but wheel locks

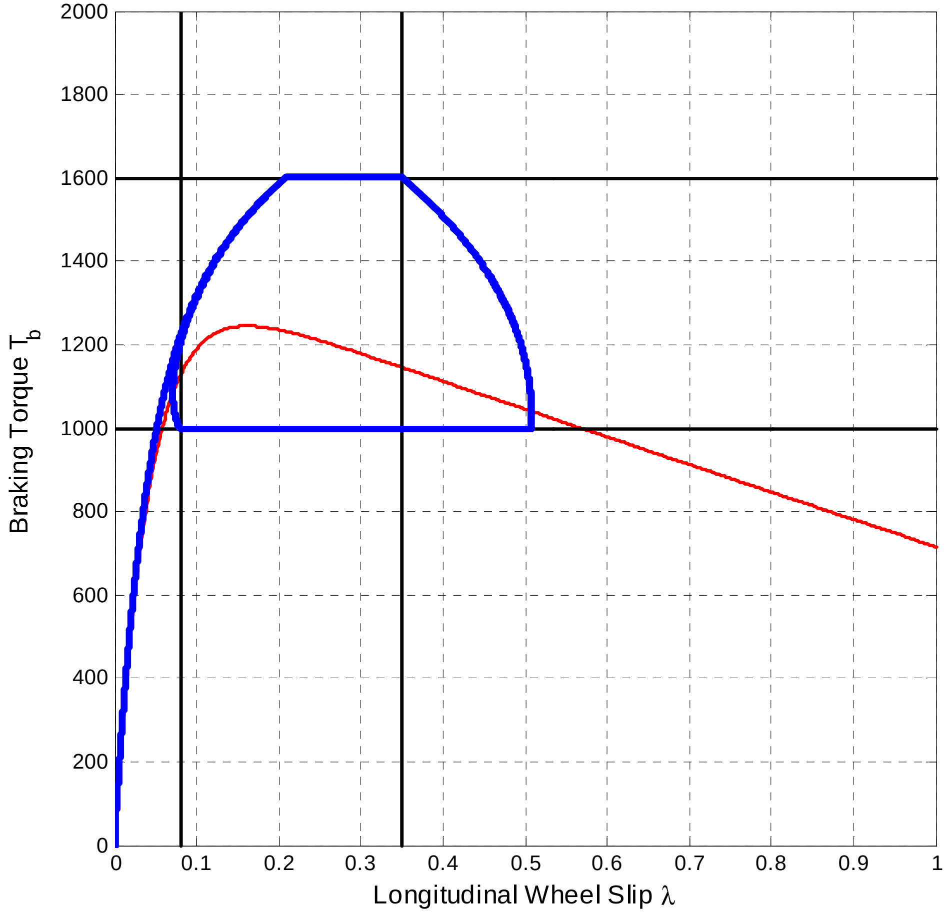


Limit-cycle: analysis

Impact of k large / small



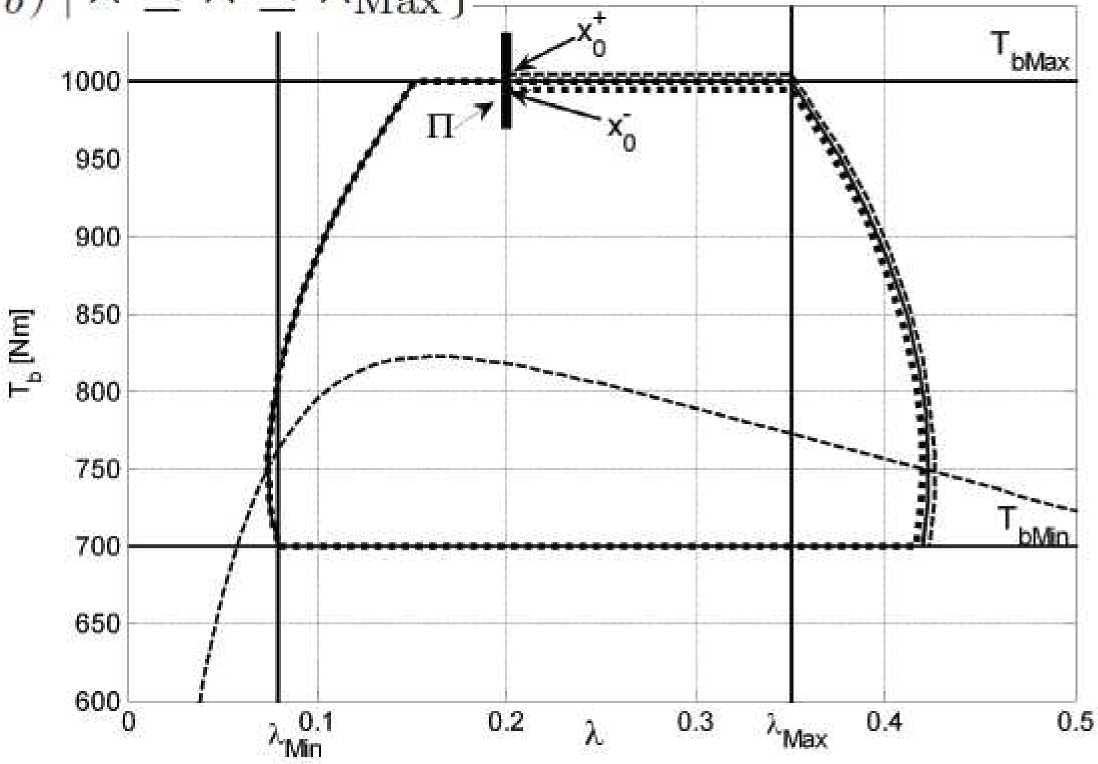
K large



K small

Limit-cycle: stability

$$\Pi := \{(\lambda, T_b) \,|\, \bar{\lambda} \leq \lambda \leq \lambda_{\mathrm{Max}}\}$$



$$x_{n+1} = P(x_n), \qquad x_n \in \mathcal{B}_r(x^*) \cap \Pi$$

$$P(x_n) = x^* \qquad \forall x_n \in \mathcal{B}_r(x^*) \cap \Pi$$

Comments

Ideal behavior:

- **"Box" very tight around the peak of the curve of friction**
- **Transitions very fast (high-k)**

In practice we have to find a compromise between:

- **k small / large**
- **Box small (performance) / large (robustness)**
- **Robustness (fixed) & Adaptation (varying) Box**