POLITECNICO DI MILANO

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Longitudinal dynamics: slip control (2)

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Outline

- Design a slip controller based on the nonlinear vehicle model

- Use a Lyapunov-based design

- The closed-loop system has different stability properties when working to the left and to the right of the friction force peak → allows detection of the working condition without additional estimators

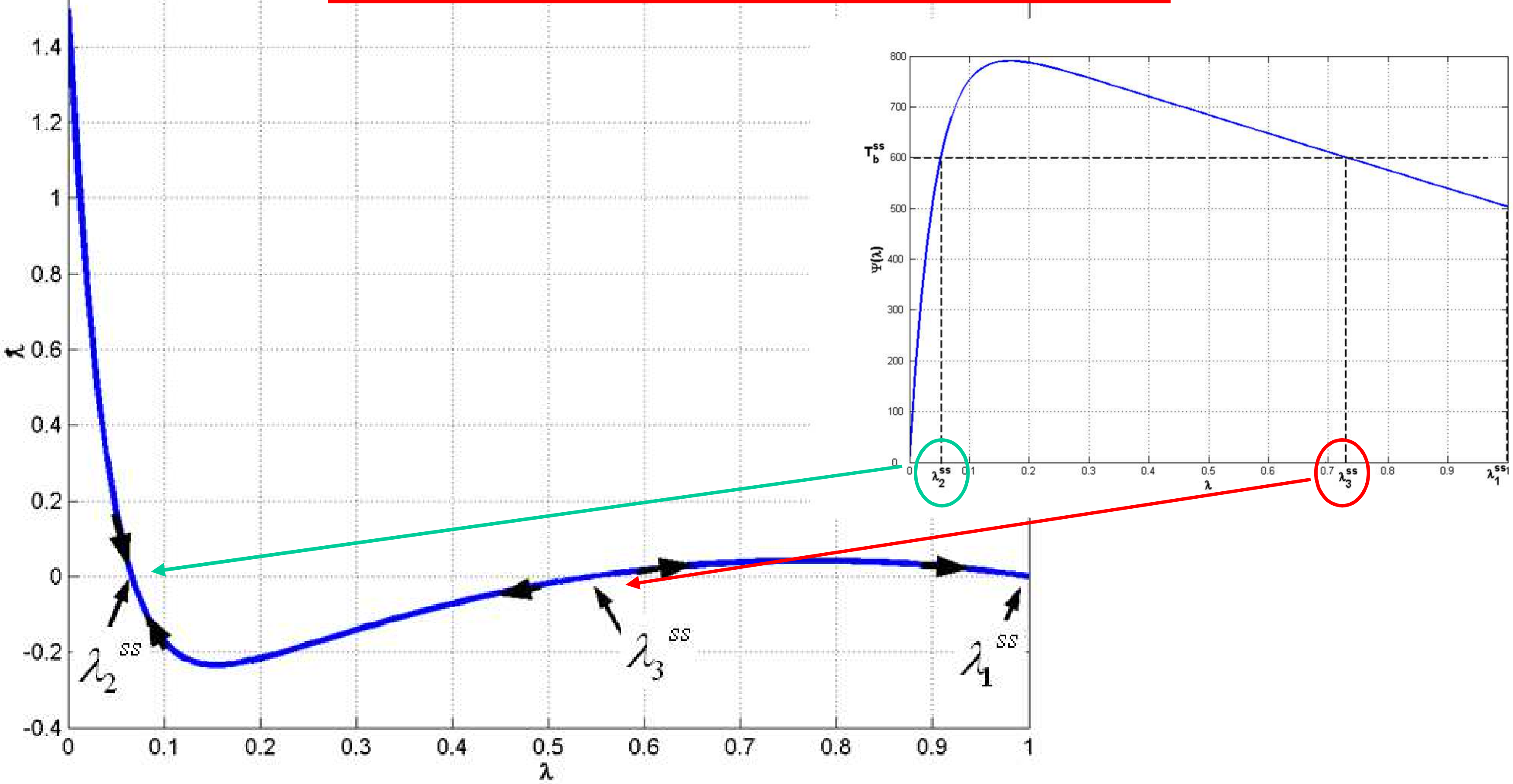
- Main references:

– M. Tanelli, A. Astolfi, S.M. Savaresi. "Robust Nonlinear Output Feedback Control for Brake-by-Wire Control Systems." Automatica. Vol. 44, No. 4, pp. 1078-1087, April 2008.

– S.M. Savaresi, M. Tanelli. Active braking control systems design for vehicles. Springer-Verlag, London, 2010.

Equilibria: stability analysis

$$\dot{\lambda} = -\frac{(1-\lambda)}{J\omega}\left(\Psi(\lambda) - T_b\right)$$



Assumptions

1. The control input T_b takes values in a non-empty subset of the real axis $T_b \in [\underline{T}_b, \overline{T}_b]$ s.t. $0 \leq \underline{T}_b < \overline{T}_b$

2. The control input T_b^* associated to the wheel slip set-point λ^* is such that $T_b^* \in [\underline{T}_b, \overline{T}_b]$ and λ^* is selected such that

$$\boxed{T_b^* < \max_{\lambda} \Psi(\lambda)}$$

3. The actuator limits $\underline{T}_b$ and $\overline{T}_b$ are such that $\underline{T}_b = 0$ and

$$\boxed{\overline{T}_b > \max_{\lambda} \Psi(\lambda)}$$

Main goal

- Design a controller in the form of a dynamic update law, such that the system is robustly stabilized around a desired equilibrium point in the sense of Lyapunov with respect to the uncertainties lying in the function $F_z\mu(\lambda)$

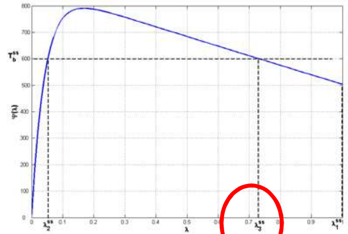
Main Result

- <u>Proposition</u>: consider the QC model

$$\boxed{\dot{\lambda} = -\frac{(1-\lambda)}{J\omega}\left(\Psi(\lambda) - T_b\right)}$$

Assume all the Ass. hold. Let $\lambda^* \in (0,1)$ be the smallest solution of $T_b^* = \max_{\lambda} \Psi(\lambda)$ and such that

$$\lambda_3^{ss}\left(1 + \frac{1}{\log\left(1-\lambda_3^{ss}\right)}\right) \leq \lambda^* < \lambda_3^{ss} \quad \text{or} \quad \lambda_3^{ss} \geq 1$$



Then, for any $\theta(0) \in (0, \overline{T}_b)$ the dynamic output feedback control law

$$\boxed{\begin{cases} T_b = & \theta \\ \dot{\theta} = & k_\lambda \frac{1}{J\omega}(\lambda - \lambda^*)(\theta - \overline{T}_b)(\theta - \underline{T}_b) \end{cases}}$$

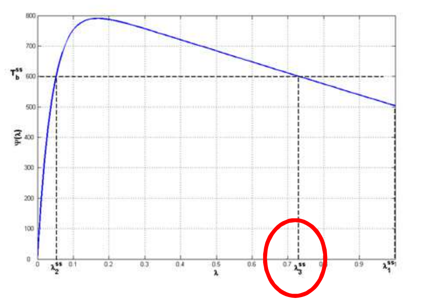
with $k_\lambda > 0,\ \omega > 0$ is such that the closed loop system is locally stable around (λ^*, θ^*) and for any initial condition $\lambda(0)$ in the region

$$\Lambda = \{\lambda \in I\!R \mid 0 \le \lambda \le 1\}$$

$\lambda(t)$ remains in this region for all t.

Moreover, if $\lambda(0) \neq 1$, $\lambda(t)$ converges asymptotically to λ^* .
Finally, the control variable T_b remains in the set $[\underline{T}_b, \overline{T}_b]$ for all $t \geq 0$.

Note: The condition on λ^* holds trivially if $\lambda^* \leq \lambda_3^{ss}$
$\lambda_3^{ss} \leq 1-e^{-1} \approx 0.6321$



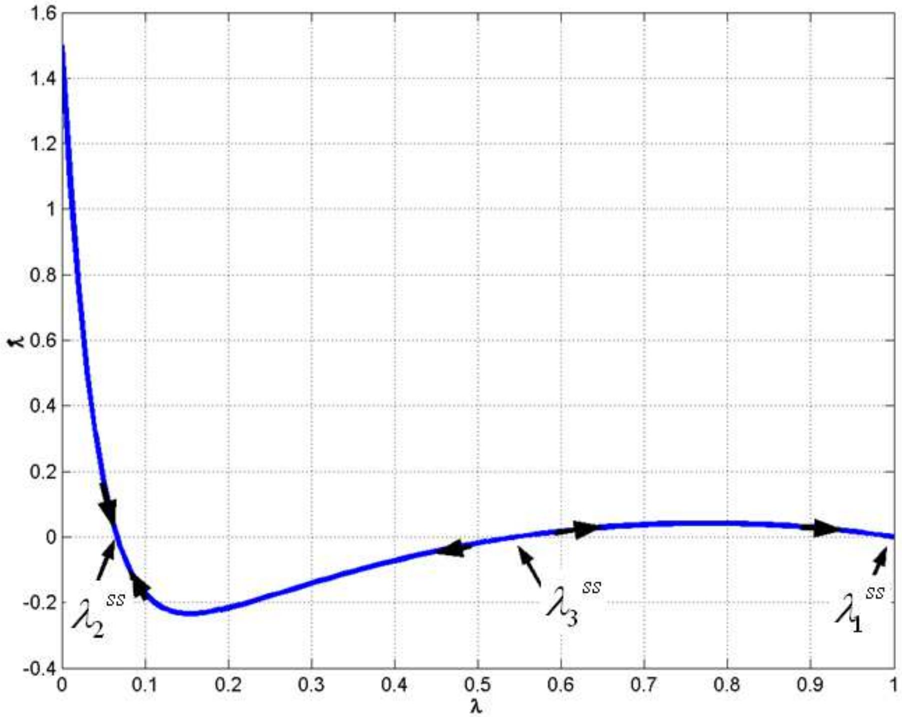
Main Result cont.d

- Let us now consider the case in which $\lambda^* \in (0,1)$ is the largest solution of
$$T_b^* = \max_{\lambda} \Psi(\lambda)$$
- In this case, the control law
$$\boxed{\begin{cases} T_b = & \theta \\ \dot{\theta} = & k_\lambda \frac{1}{J\omega}(\lambda - \lambda^*)(\theta - \overline{T}_b)(\theta - \underline{T}_b) \end{cases}}$$
is such that, for any initial condition $\lambda(0) \in \Lambda$, λ(t) remains in this region for all t.
- If λ(0)≠ 1, the closed-loop trajectory (λ(t),θ(t)) converges to an attractive periodic orbit encircling the equilibrium (λ^*, θ^*)
- Again, the control variable T_b remains in the set $[\underline{T}_b, \overline{T}_b]$ for all t≥0 .

Main Result: proof

$$\Lambda = \{\lambda \in \mathbb{R} \mid 0 \leq \lambda \leq 1\}$$

From the vector field behavior and the definition of λ we can see that the region Λ is invariant



Main Result: proof cont.d

- Consider the candidate Lyapunov function

$$W(\lambda,\theta) = V(\lambda) + \xi(\theta) + c$$

where

$$\boxed{V(\lambda) = -\lambda + (\lambda^* - 1)\ln(1-\lambda)}$$

$$\boxed{\xi(\theta) := \ln(\overline{T}_b - \theta)^{\tau/k_\lambda} - \ln(\theta - \underline{T}_b)^{(\tau+1)/k_\lambda}}$$

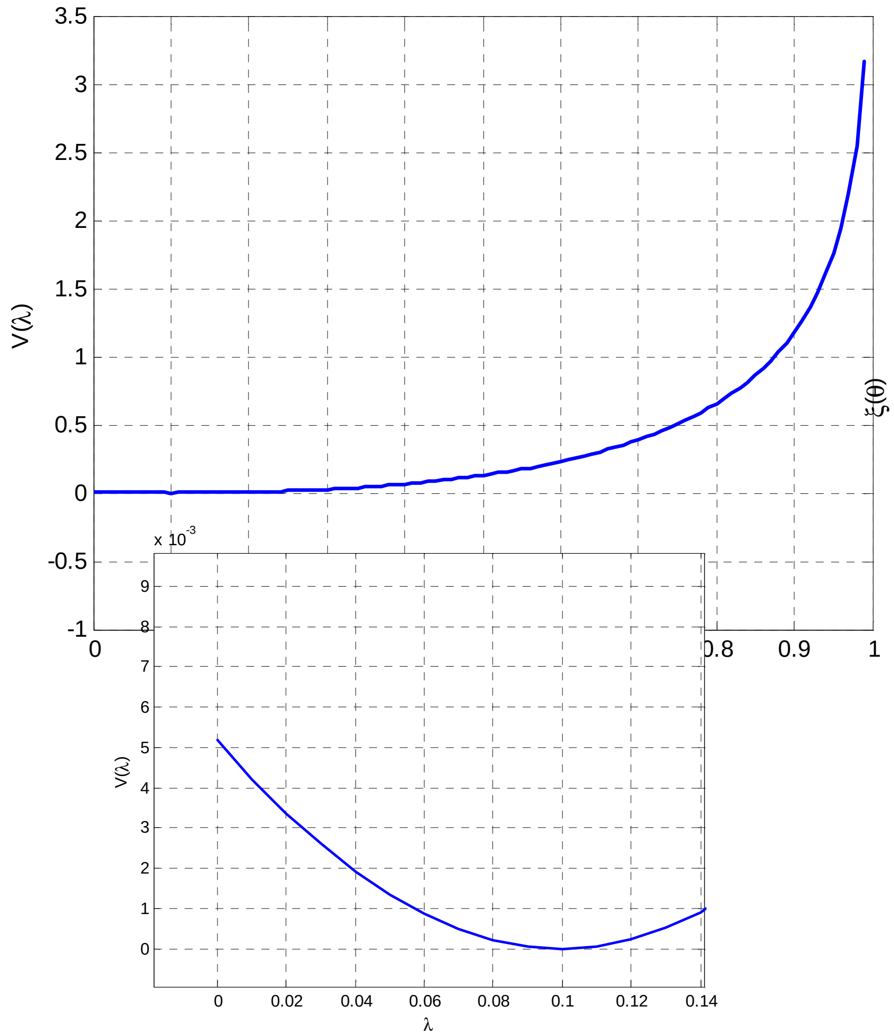
with $\tau = \dfrac{T_b^* - \overline{T}_b}{\overline{T}_b - \underline{T}_b} < 0$ and $\tau + 1 > 0$

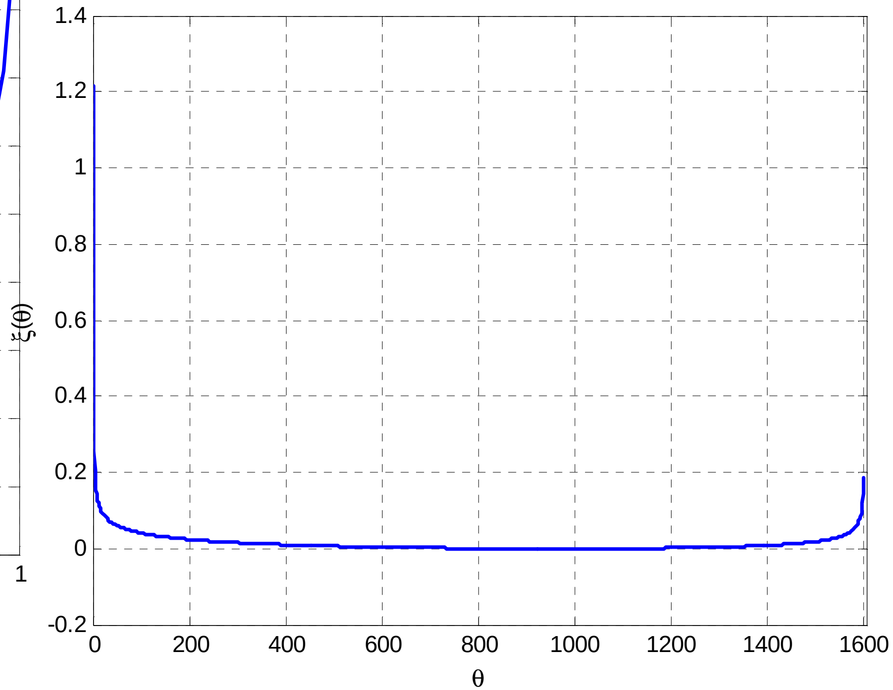
c is chosen such that $W(\lambda^*, T_b^*) = 0$

Main Result: proof cont.d

W(λ,θ) is positive definite in

$$\boxed{(\lambda,\theta) \in (0,1) \times (\underline{T}_b, \overline{T}_b)}$$





Main Result: proof cont.d

- Now note that

$$
\begin{aligned}
\dot{W} &= \frac{\partial V}{\partial \lambda}\dot{\lambda} + \frac{\partial \xi}{\partial \theta}\dot{\theta} = -\left[\frac{(\lambda^* - 1)}{(1-\lambda)} + 1\right]\dot{\lambda} + \frac{\partial \xi}{\partial \theta}\dot{\theta} = \\
&= -\tfrac{(\lambda^*-\lambda)}{(1-\lambda)}\left\{-\tfrac{1-\lambda}{\omega r}\left(\tfrac{(1-\lambda)}{m} + \tfrac{r^2}{J}\right)F_z\mu(\lambda) + \tfrac{r}{J}\tfrac{1-\lambda}{\omega r}T_b^*\right\} + \\
&+ (\lambda - \lambda^*)\tfrac{1}{J\omega}(T_b - T_b^*) + \frac{\partial \xi}{\partial \theta}\dot{\theta} = \\
&= -\tfrac{(\lambda^*-\lambda)}{(1-\lambda)}\left\{-\tfrac{1-\lambda}{\omega r}\left(\tfrac{(1-\lambda)}{m} + \tfrac{r^2}{J}\right)F_z\mu(\lambda) + \tfrac{r}{J}\tfrac{1-\lambda}{\omega r}T_b^*\right\} + \\
&+ (\lambda - \lambda^*)\tfrac{1}{J\omega}\left[(T_b - T_b^*) + \frac{\partial \xi}{\partial \theta}\sigma(\theta)\right].
\end{aligned}
$$

Main Result: proof cont.d

- Recalling the definition of $\Psi(\lambda)$

$$\boxed{\begin{aligned} \dot{W} &= -\tfrac{(\lambda-\lambda^*)}{J\omega}\left\{\Psi(\lambda) - T_b^*\right\} + \\ &+ (\lambda-\lambda^*)\tfrac{1}{J\omega}\left[(T_b - T_b^*) + \frac{\partial \xi}{\partial \theta}\sigma(\theta)\right] \end{aligned}}$$

- Given the expression of $\xi(\theta)$, if we choose

$$\sigma(\theta) = k_\lambda(\theta - \overline{T}_b)(\theta - \underline{T}_b)$$

$$(T_b - T_b^*) + \frac{\partial \xi}{\partial \theta}\sigma(\theta) = 0$$

Main Result: proof cont.d

- Hence, in view of the Ass., $(\lambda-\lambda^*)$ and $(\Psi(\lambda) - T_b^*)$ have the same sign for all $\lambda 2(0, \lambda_3^{ss})$

their product can be written as $(\lambda-\lambda^*)^2\Xi(\lambda)$, with $\Xi(\lambda)>0$ for all $\lambda\in(0, \lambda_3^{ss})$

$$\boxed{\dot{W} = -(\lambda - \lambda^*)^2\Xi(\lambda) \leq 0}$$

for all $\lambda\in(0, \lambda_3^{ss}]$ and $\theta \in (\underline{T}_b, \overline{T}_b)$

Main Result: proof cont.d

- Hence, as $\lambda^* < \lambda_3^{ss}$ we have shown that the equilibrium point (λ^*,θ^*) is locally stable

- To complete the proof we have to show that any trajectory of the closed-loop system is such that $\theta(0) \in (0, \overline{T}_b)$ and $\lambda(0) \in [0,1)$ converges to this equilibrium

- The system is planar (2D) $\rightarrow$ the only possible attractors are zero or one-dimensional (*i.e.,* equilibria or limit cycles)

Main Result: proof cont.d

- Closed-loop dynamics:

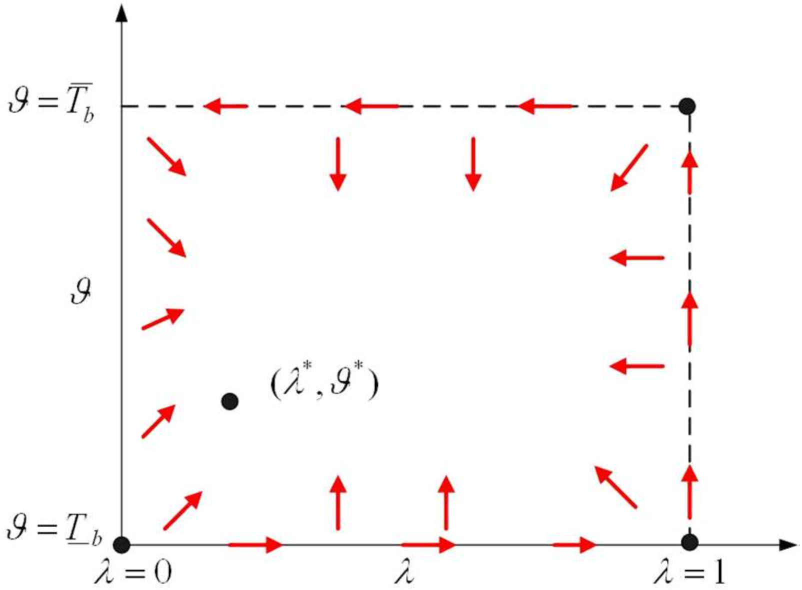
$$\begin{cases} \dot{\lambda} = & -\frac{1-\lambda}{J\omega}(\Psi(\lambda)-\theta) \\ \dot{\theta} = & k_\lambda \frac{1}{J\omega}(\lambda-\lambda^*)(\theta-\overline{T}_b)(\theta-\underline{T}_b) \end{cases}$$

- In view of the assumptions,we need to study

i) $\lambda = 1$ and $\theta = \overline{T}_b$;

ii) $\lambda = 1$ and $\theta = \underline{T}_b = 0$;

iii) $\lambda = 0$ and $\theta = \underline{T}_b = 0$.



Main Result: proof cont.d

- If we define $\delta\lambda := \lambda - \lambda^{ss}$ $\delta\theta := \theta - \theta^{ss}$nd

$$\mu_1(\lambda) := \frac{d}{d\lambda}\mu(\lambda)\Big|_{\lambda=\lambda^{ss}}$$

- The linearized system is given by

$$\begin{cases} \dot{\delta\lambda} = & \{-rF_z\left[-\mu(\lambda^{ss}) + (1-\lambda^{ss})\mu_1(\lambda^{ss})\right] \\ & -\frac{JF_z}{m}\left[-2(1-\lambda^{ss})\mu(\lambda^{ss}) + (1-\lambda^{ss})^2\mu_1(\lambda^{ss})\right] \\ & -\theta^{ss} + (1-\lambda^{ss})\theta^{ss}\}\,\delta\lambda \\ & +(1-\lambda^{ss})\delta\theta \\ \dot{\delta\theta} = & -k_\lambda\lambda^*(\theta^{ss} - \overline{T}_b)(\theta^{ss} - \underline{T}_b)\delta\lambda \\ & +k_\lambda(\lambda^{ss} - \lambda^*)\left[2\theta^{ss} - (\underline{T}_b + \overline{T}_b)\right]\delta\theta \end{cases}$$

Main Result: proof cont.d

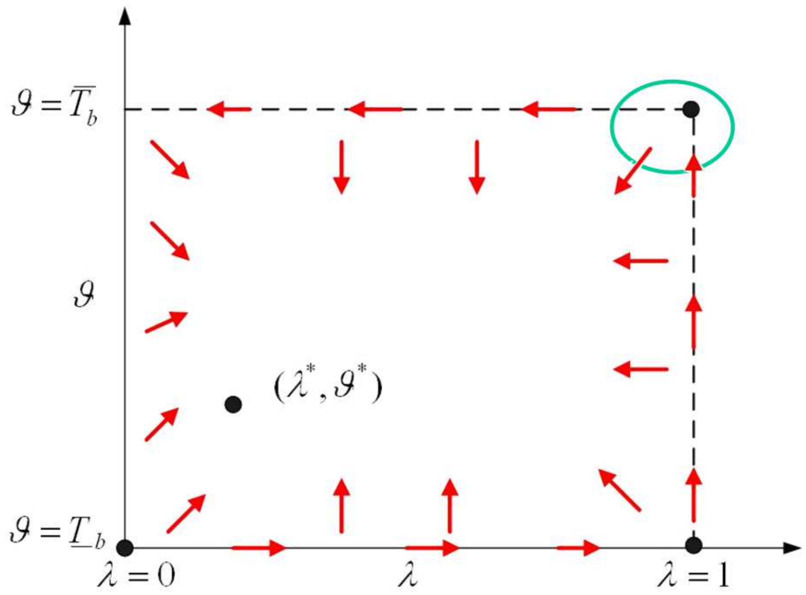
- Accordingly

$$A_{(1,\overline{T}_b)} = \begin{bmatrix} rF_z\mu(1) - \overline{T}_b & 0 \\ 0 & k_\lambda(1-\lambda^*)[\overline{T}_b - \underline{T}_b] \end{bmatrix}$$

$(1, \overline{T}_b)$ is a saddle point

the only initial conditions yielding trajectories that converge to this equilibrium are such that

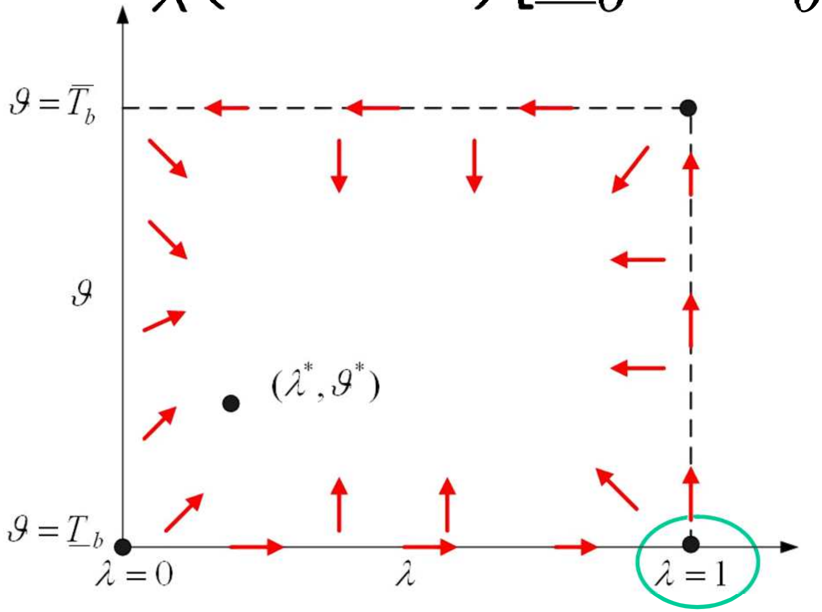
$$\lambda(0) = 1$$



Main Result: proof cont.d

- Then we have

$$A_{(1,\underline{T}_b)} = \begin{bmatrix} rF_z\mu(1) - \underline{T}_b & 0 \\ 0 & k_\lambda(1-\lambda^*)[\underline{T}_b - \overline{T}_b] \end{bmatrix}$$



$(1, \underline{T}_b)$ is a saddle point

the only initial conditions yielding trajectories that converge to this equilibrium are such that $\theta(0) = 0$

Main Result: proof cont.d

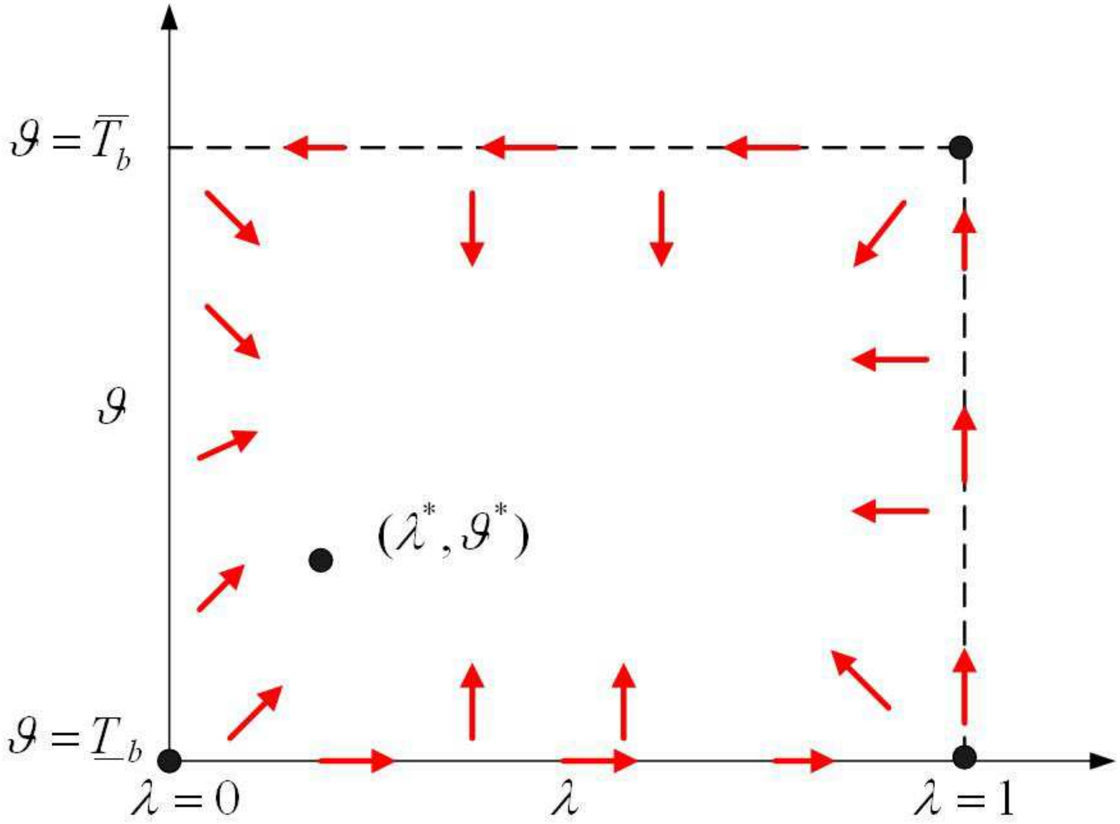
- Finally

$$A_{(0,\underline{T}_b)} = \begin{bmatrix} rF_z\mu(0) - \underline{T}_b & 0 \\ 0 & -k_\lambda \lambda^* [\underline{T}_b - \overline{T}_b] \end{bmatrix}$$

$$\Downarrow$$

$(0, \underline{T}_b)$ is an unstable saddle-node point

Main Result: proof cont.d



$$\mathcal{D} = \left\{ (\lambda, \theta) \in I\!R^2 \mid 0 \leq \lambda < 1, \theta \in (\underline{T}_b, \overline{T}_b) \right\}$$

Main Result: proof cont.d

- To show that all trajectories with initial condition in $\mathcal{D}$ do converge to (λ^*, θ^*) we only need to show that there are no limit cycles in this region

- However, the only limit cycle that may exist should encircle (λ^*, θ^*)

Main Result: proof cont.d

- Consider the level set

$$\Omega = \{(\lambda, \theta) \in \bar{\mathcal{D}} \mid W(\lambda, \theta) \leq W(0, \theta^*)\}$$

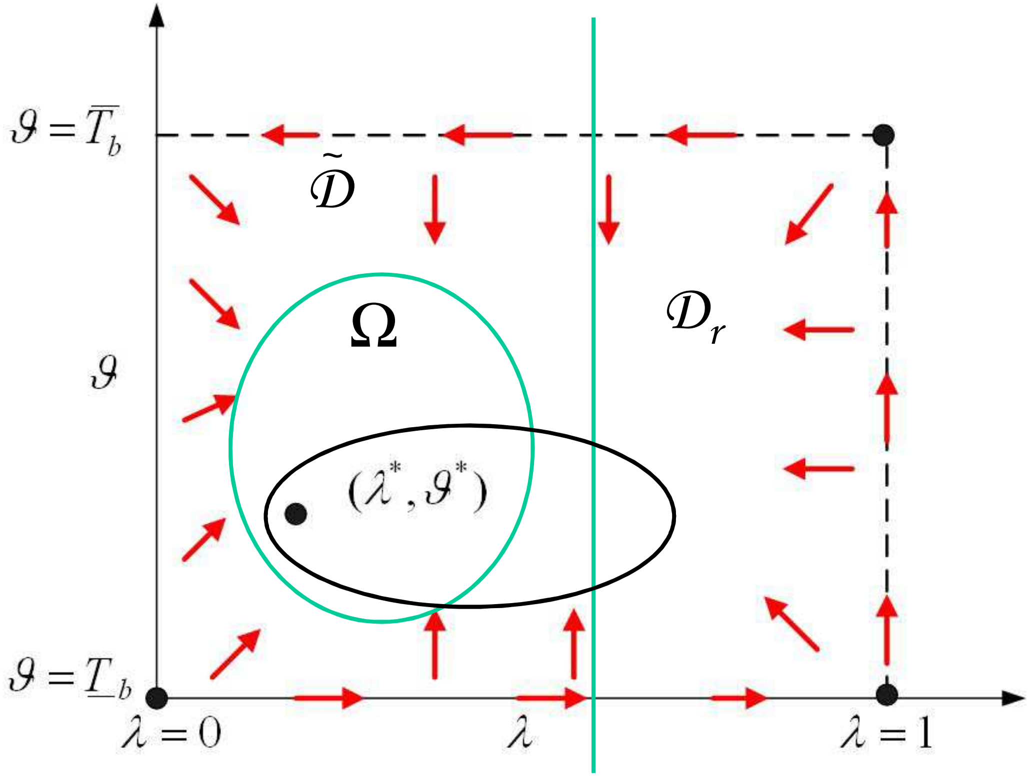
- This is contained in the set

$$\tilde{\mathcal{D}} = \{(\lambda, \theta) \in \bar{\mathcal{D}} \mid \lambda \leq \lambda_3^{ss}\}$$

- Hence, given the expression of $\dot{W}$ any limit cycle around (λ^*, θ^*) should be contained in

$$\mathcal{D}_r = \mathcal{D} \setminus \Omega$$

Main Result: proof cont.d



However, it is impossible to construct a closed curve contained in $\mathcal{D}_r$ and encircling the point (λ^*,θ^*)

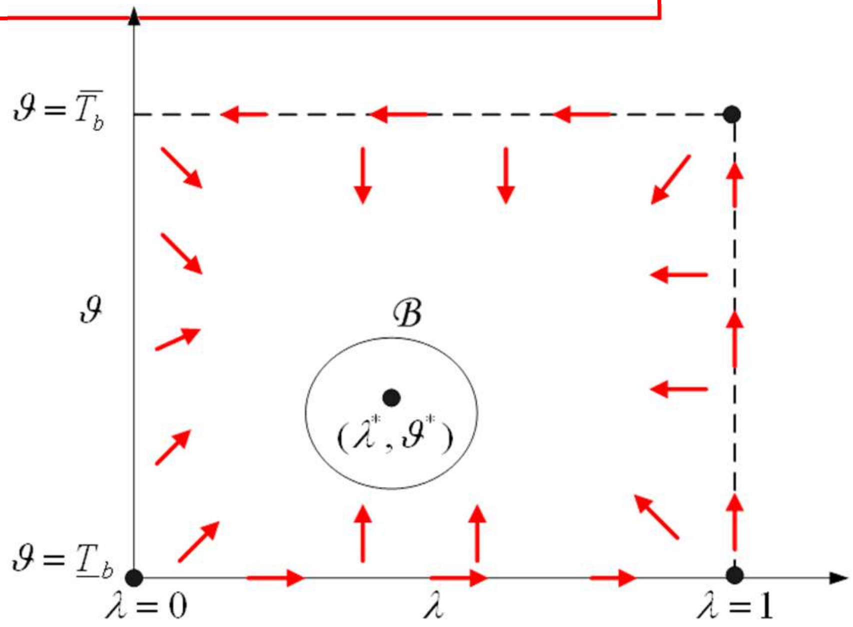
As a consequence, all trajectories starting in $\mathcal{D}$ will converge to the equilibrium (λ^*,θ^*)

Main Result: proof cont.d

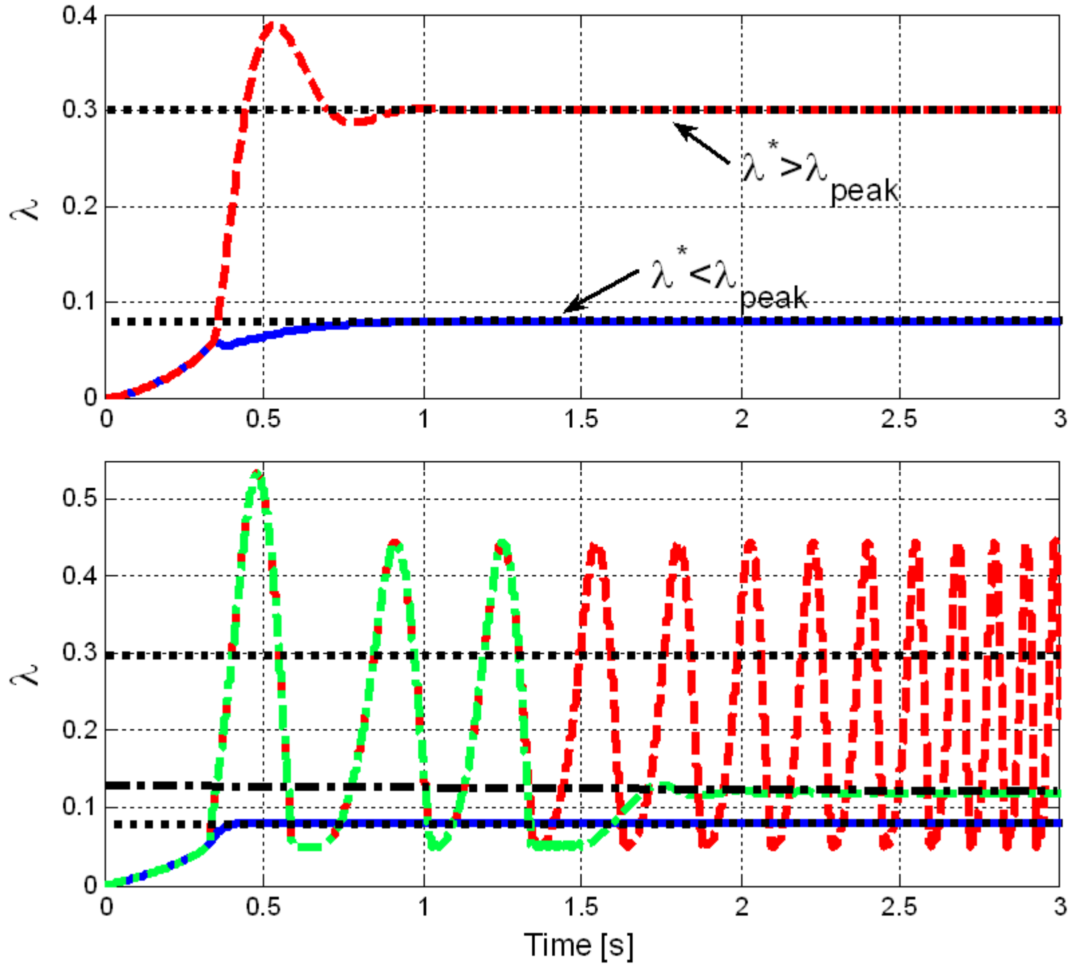
- Let us study the case when λ^* is the largest solution of $T_b^* = \max_\lambda \Psi(\lambda)$ (λ*,θ*) is an unstable equilibrium $(\dot{W} \geq 0)$
- Consider a small neighborhood B of (λ*,θ*) and the set difference $\mathcal{C} = \mathcal{D} \setminus B$

As (λ*,θ*) is unstable, all trajectories starting in B will converge to C and stay in the set. As C does not contain any equilibrium, Poincarè-Bendixon theorem guarantees that these trajectories either are periodic orbits or tend to periodic orbits.

Again, the instability of (λ*,θ*) ensures that (Index Theory) the periodic orbit is unique → any trajectory in C converges to an attractive periodic orbit (the ω-limit set of the vector field is nonempty for the positive invariance of $\mathcal{D}$)

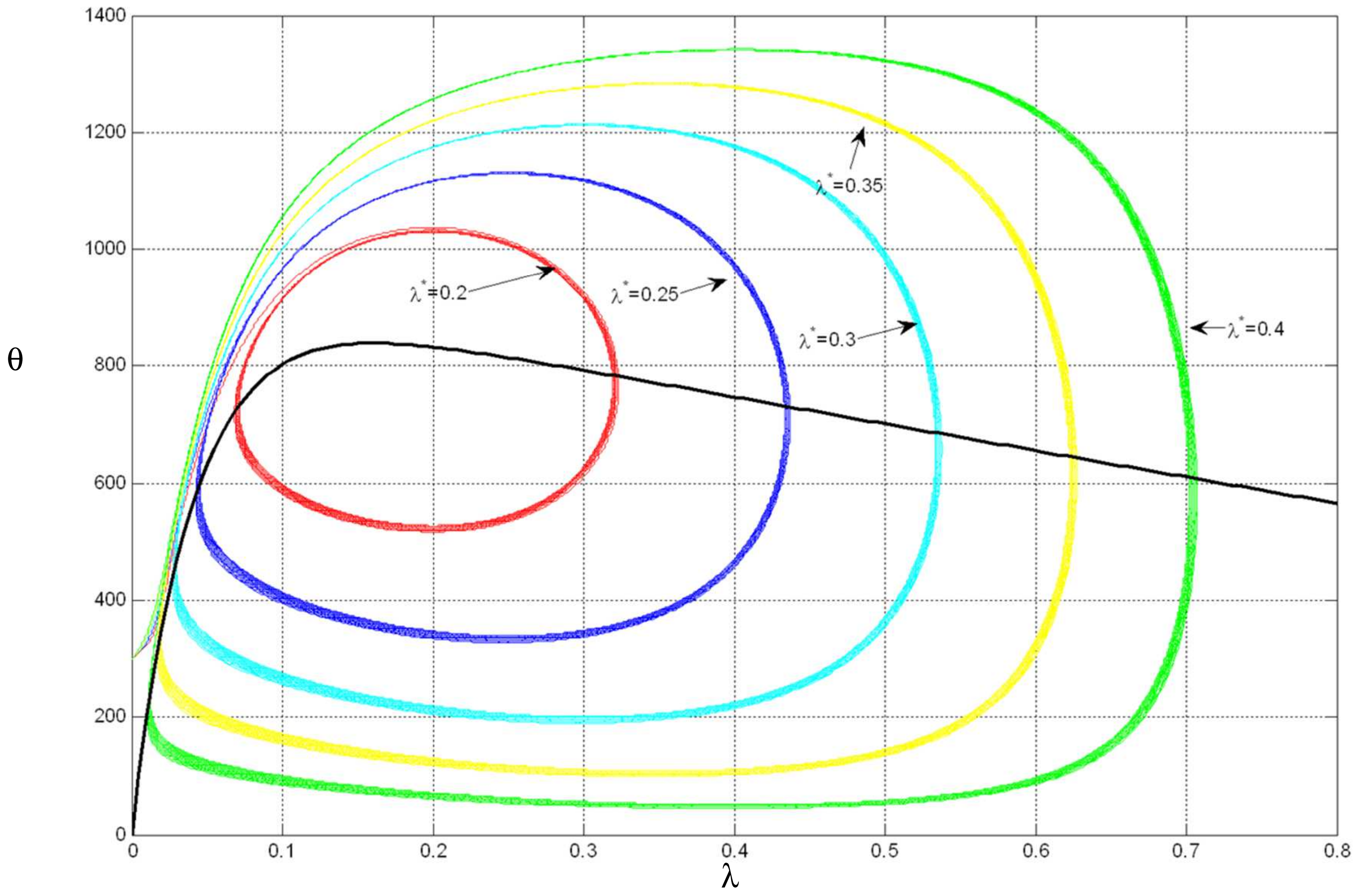


Main advantage



The proposed control allows to detect in which region of the friction curve the system is operating!

Main advantage (2)



Main advantage (3)

The proposed control allows adapting the set-point to maximize the friction force with little computational effort